Generalized coherent calibration using small area estimates

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Table of contents

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The challenge of coherent estimation

Principle 14 of the *European Statstics Code of Practice* recommends *coherence and comparability* of statistics. The following kinds of coherence shall be considered:

- Internal coherence
- Coherence between regions and by subject (tables)
- Coherence over time
- Coherence with respect to definitions and surveys

Census 2011 Estimation at different regional levels, likely with different methods

New integrated household surveys Estimates of the master sample (Germany: microcensus) versus additional surveys (LFS, SILC, ...)

The challenge of coherent estimation

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Problem of coherent census estimates

- Core estimates

  Goal 1 GREG estimates

  Goal 2 (NUTS3) GREG preferred

  Goal 2 (LAU) GREG likely to be inaccurate: SAE
- Eurostat hypercubes:
  - Overlap of parts of marginals possible
  - Different estimation methods may be optimal
  - ... *are likely to be incoherent*
- Many estimates on different levels

The aim of the German Federal Statistical Office is to gain coherent estimates, preferably via one *vector of weights*: **one number census**!

Census, weights, and estimation

- The German register-assisted census is drawn via box-constraint optimal allocation which allows to include minimal and maximal sampling fractions
- This allows to constrain the variation of weights (here: 25) referring to the critique of Gelman (2007)
- However, the weights also have to be considered for small area estimation
- Negative or extreme weights shall be *cut*
- GREG and calibration-based estimators allow adequate accuracy estimates even if possible model-assumptions are violated (part of the German census law)

Generalized calibration with penalties (cf. Münnich, Sachs and Wagner, 2011) allows coherent benchmarking with small area estimates

Benchmark for the census I

- Goal 1: Combined GREG for relevant regions
  $\Rightarrow$ exact control (Condition I)
- Goal 2: Combined GREG on NUTS3
  $\Rightarrow$ little (or no) tolerance (Condition IIa)
  (alternative estimates are possible)
- Goal 2: You/Rao estimator on LAU-level
  $\Rightarrow$ larger tolerance needed (Condition IIb)

Note: Tolerated perturbation depends on the importance of the auxiliary variable for the census estimates. The solution (including weight variation control) can be obtained using complex solvers but has very large and sparse design matrices and suffers from zigzagging effects.

Benchmark conditions for the census II

- Due to the numerical problems at the boundaries (non-differentiable areas), the algorithm had to be extended using semi-smooth Newton methods
- Additionally: too large deviations from the registers to the final estimates on goal 1 (subgroups in subregions) urged the need for adding further constraints
  additional constraint on AGE x GEN for goal 1 (condition III)

The methodology must allow an easy and sophisticated control of the efficacy of the different calibration constraints that enables the user to set the (needed) tolerances individually!

Generalized calibration using penalties

$$\min_{(g,\epsilon^{I}_{KRS},\epsilon^{I}_{SMP},\epsilon^{II})} \sum_{k\in s} d_k \frac{(g_k-1)^2}{2} + \sum_{k\in I} \delta_k^{KRS} \frac{(\epsilon^{I}_{KRS_k}-1)^2}{2} + \sum_{k\in J} \delta_k^{SMP} \frac{(\epsilon^{I}_{SMP_k}-1)^2}{2} + \sum_{k\in K} \gamma_k \frac{(\epsilon^{II}_k-1)^2}{2}$$

$$\begin{aligned}
s.t.\ & \hat{\tau}^{CAL}_{SMP,ZEN} := X^{CAL}_{I,SMP,ZEN} \cdot g = \hat{\tau}^{GREG}_{SMP,ZEN} \\
& \hat{\tau}^{CAL}_{KRS,Cal} := X^{CAL}_{IIa,KRS,Cal} \cdot g = diag(\hat{\tau}^{YR}_{KRS,Cal}) \cdot \epsilon^{I}_{KRS} \\
& \hat{\tau}^{CAL}_{SMP,Cal} := X^{CAL}_{IIb,SMP,Cal} \cdot g = diag(\hat{\tau}^{YR}_{SMP,Cal}) \cdot \epsilon^{I}_{SMP} \\
& \tau^{CAL}_{KRS,A\times G} := \hat{X}^{CAL}_{III,KRS,A\times G} \cdot g = diag(\tau^{\mathbf{REG}}_{KRS,A\times G}) \cdot \epsilon^{II} \\
& g \in \Omega \\
& \epsilon^{I}_{KRS} \in \Omega^{I}_{KRS} \\
& \epsilon^{I}_{SMP} \in \Omega^{I}_{SMP} \\
& \epsilon^{II} \in \Omega^{II}
\end{aligned}$$

The solution is obtained via *semi-smooth Newton calibration* (cf. Münnich, Sachs, and Wagner, 2011).

Simulation study - Overview

Census of Rhineland-Palatinate and Saarland:

- Goal 1 restrictions on SMP level
- Goal 2 restrictions on KRS level: e.g. EF117 classes
  $\Rightarrow$ Permitted tolerance per KRS: ϵ^{I}_{KRS}
- Goal 2 restrictions on KRS level: e.g. EF117 classes
  $\Rightarrow$ Permitted tolerance per SMP: ϵ^{I}_{SMP}
- Age $\times$ Gender classes:
  $\Rightarrow$ Permitted tolerance per SMP: ϵ^{II}
- Box-Constraints for calibration weights g
- Box-Constraints for deviation of ϵ^{I}_{KRS}, ϵ^{I}_{SMP} and ϵ^{II}

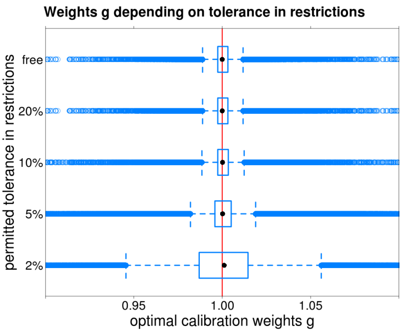
Performance of semi-smooth Newton method

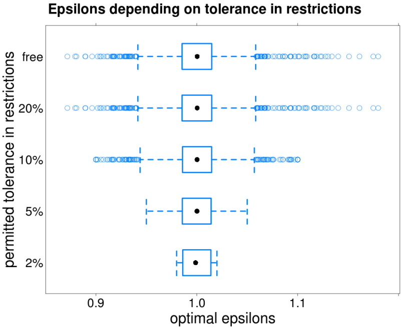
- **Compare** performance of
  1. **Semi-smooth Newton method** (SSN)
  2. Classical **truncated** calibration (TRUNC)
     (modified 'calib' function from R Package 'sampling')
- **Scenarios 1-5:** Tolerance for AxG decreases from *free* to 2%

Tolerance for AxG	Norm of constraints (reached optimum, if $< tol$)		Value of objective (the lower the better)	
	SSN	TRUNC	SSN	TRUNC
free	$2.13 \cdot 10^{-9}$	$3.85 \cdot 10^{-8}$	59.24	59.24
20%	$2.13 \cdot 10^{-9}$	$3.85 \cdot 10^{-8}$	59.24	59.24
10%	$1.99 \cdot 10^{-10}$	$1.31 \cdot 10^{-7}$	102.11	103.23
5%	$3.32 \cdot 10^{-10}$	$4.35 \cdot 10^{-7}$	495.72	517.00
2%	$1.23 \cdot 10^{-9}$	$3.49 \cdot 10^{+2}$	1653.45	1747.27

Distribution of weights and deviation from benchmarks

- **Scenarios 1-5:** Tolerance for AxG decreases from *free* to 2%



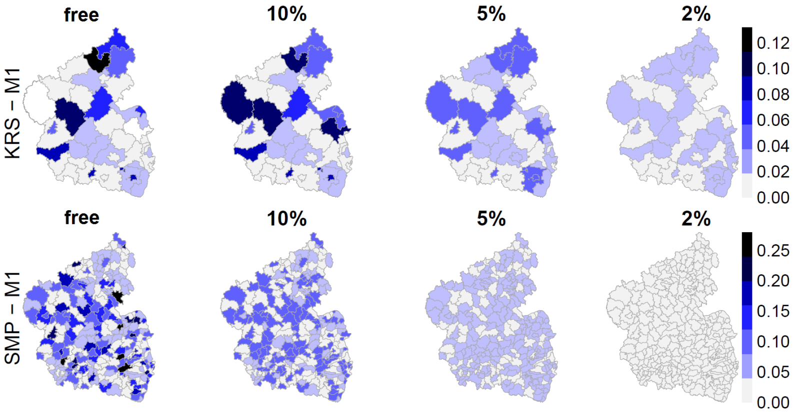


- Variance of weights *increases* while tolerance *decreases*

- Deviations from the benchmarks are pushed into the box of given tolerance

Deviation of AxG classes from the benchmark (register)

- **Scenarios 1-5:** Tolerance for AxG decreases from *free* to 2%
- Example class: male and age < 20



Summary and outlook

- Generalized calibration with flexible penalties
  - Is a very flexible tool in survey practice considering model estimates (incl. model and hybrid calibration)
  - Allows easily to add soft and hard constraints
  - Enables post-editing and evaluation in terms of areas, efficacy of constraints, variables and their outcomes
- Variance estimation
  - Rescaling Bootstrap
  - Use special linearization as variance approximation
- Semi-smooth Newton algorithm is essential
- R-Package (easy to use with examples)
- Extension to household surveys: Achieve coherence between different surveys (e.g. LFS and SILC)

Thank you for your attention!

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