A New Version of the Item Count Technique

Eleni Manoli and Tasos C. Christofides

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1. Introduction

Classical survey procedures are not appropriate for the investigation of sensitive items. People are reluctant to participate in surveys dealing with matters of privacy, and people participating often provide untruthful responses.
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Stigmatizing Issues

Sexual Behavior

Drug Abuse

Political affiliation (in certain societies)

Engagement in criminal activities

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- Engagement in criminal activities
Indirect Questioning Techniques

Indirect questioning has been used by psychologists and merchants since the 1950s. Warner (1965) was the first researcher who published a research paper concerning Indirect Questioning Techniques. These techniques allow the respondent to provide an answer from which it is not possible for the interviewer to infer whether the specific respondent belongs to the stigmatizing category.
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**Protection of Privacy**
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Item Count Technique (ICT)

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- Estimate the proportion $\theta$ of people belonging to the stigmatizing category
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Estimate the proportion $\theta$ of people belonging to the stigmatizing category

Two lists are used:

a) the long item list which includes $G + 1$ items ($G$ are non sensitive and one is sensitive) and

b) the short item list which includes the $G$ non sensitive items
The Item Count Technique

In both samples, the respondents should report the total number of items that apply to them without disclosing which ones.

Let $X_i$ be the number reported by person $i$ from the first sample ($i = 1, \ldots, n_1$) and let $Y_j$ be the number reported by person $j$ from the second sample ($j = 1, \ldots, n_2$). Then $\hat{\theta} = \bar{X} - \bar{Y}$ is an estimator of $\theta$.
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What follows are sample lists which can be used to estimate the prevalence of illegal doping among professional athletes.

<table>
<thead>
<tr>
<th>Questionnaire 1</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Statement</strong></td>
</tr>
<tr>
<td>1 I am on a high protein diet.</td>
</tr>
<tr>
<td>2 My mother was/is allergic to fish.</td>
</tr>
<tr>
<td>3 I make use of illegal doping.</td>
</tr>
<tr>
<td>4 I have taken antibiotics during last year.</td>
</tr>
<tr>
<td>5 I have never been hospitalized.</td>
</tr>
<tr>
<td>6 Before I became a professional athlete I used to take vitamins on a daily basis.</td>
</tr>
<tr>
<td>7 After retirement, I will become a trainer for professional athletes.</td>
</tr>
</tbody>
</table>

*Total Score:*
<table>
<thead>
<tr>
<th>a/a</th>
<th>Statement</th>
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</tr>
</thead>
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*Total Score:*
The Item Count Technique

Disadvantages:

a) Only the treatment group gives information on the sensitive characteristic we are investigating

b) In cases where the interviewee reports the number $G + 1$ as his/her item score, then the interviewer can be sure that the respondent has the sensitive item
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  b) In cases where the interviewee reports the number G + 1 as his/her item score, then the interviewer can be sure that the respondent has the sensitive item

Advantages:
  a) It can be incorporated into regular questionnaires

  b) It does not require respondents to conduct a randomization experiment
Purpose of the new version: To protect the privacy of the participants and to estimate the population proportion $\theta$ of people who belong to the sensitive category.
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How is this achieved?
An improved version of the Item Count Technique

Purpose of the new version: To **protect the privacy** of the participants and to estimate the population proportion $\theta$ of people who belong to the sensitive category.

How is this achieved? The interviewer **cannot infer** from any response that the participant does or does not belong to the stigmatizing category.
2. The method

Two independent random samples of sizes $n_1$ and $n_2$ are drawn with replacement from the population. First Sample:

- Every participant is presented with a list of $G+1$ items of which $G$ are non-sensitive and one is sensitive.
- He/she has to study the items and count how many of them are applicable to him or her without disclosing which ones.
- If all $G+1$ items are applicable, then he or she must report the number 1; if zero items are applicable, then the participant must report the number $G$. In any other case, he/she must report the exact number of the applicable items.

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A New Version of the Item Count Technique
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  ✓ He/she has to study the items and to count how many of them are applicable to him or her without disclosing which ones
  
  ✓ He/she must report the exact number of applicable items
2. The method

Let $q_k$ be the probability that an individual from the first sample reports the number $k$, $k = 1, \ldots, G$.

Let $p_k$ be the probability that exactly $k$ of the non-stigmatizing items are applicable to an individual, $k = 0, 1, \ldots, G$.

Then

\begin{align*}
q_1 &= p_0 \theta + p_1 (1 - \theta) + p_G \theta, \\
q_k &= p_k - 1 \theta + p_k (1 - \theta), \quad \text{for } k = 2, \ldots, G - 1, \\
q_G &= p_{G-1} \theta + p_G (1 - \theta) + p_0 (1 - \theta). 
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- Then

$$q_1 = p_0 \theta + p_1 (1 - \theta) + p_G \theta,$$  \hspace{1cm} (1)

$$q_k = p_{k-1} \theta + p_k (1 - \theta), \text{ for } k = 2, \ldots, G - 1,$$  \hspace{1cm} (2)

$$q_G = p_{G-1} \theta + p_G (1 - \theta) + p_0 (1 - \theta)$$  \hspace{1cm} (3)
Let $n_k^{(1)}$ be the number of individuals from the first sample reporting the number $k$, $k = 1, \ldots, G$.
• Let \( n_k^{(1)} \) be the number of individuals from the first sample reporting the number \( k \), \( k = 1, \ldots, G \)

• Then the vector \( (n_1^{(1)}, \ldots, n_G^{(1)}) \) follows the multinomial distribution with parameters \( n_1, q_1, \ldots, q_G \)
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Let $n_k^{(2)}$ be the number of individuals from the second sample reporting the number $k$, $k = 0, \ldots, G$
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Let $n_k^{(2)}$ be the number of individuals from the second sample reporting the number $k$, $k = 0, \ldots, G$.

Then the vector $\left( n_0^{(2)}, \ldots, n_G^{(2)} \right)$ follows the multinomial distribution with parameters $n_2, p_0, \ldots, p_G$. 

Consider the following quantities:

\[ C_1 = \frac{1}{n_1} \sum_{k=1}^{G} k n_k^{(1)} \]

\[ C_2 = \frac{1}{n_2} \sum_{k=1}^{G} k n_k^{(2)} \]

Using properties of the multinomial distribution it can be shown that

\[ \theta = E(C_1 - C_2) - G p_0 - G(p_G + p_0) \]

assuming that

\[ G(p_G + p_0) \neq 1 \]
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Using properties of the multinomial distribution it can be shown that

\[
\theta = \frac{E(C_1 - C_2) - Gp_0}{1 - G(p_G + p_0)},
\]  \hspace{1cm} (4)

assuming that \( G(p_G + p_0) \neq 1 \)
Equation (4) suggests as an estimator for the parameter $\theta$ the quantity

$$\hat{\theta} = \frac{C_1 - C_2 - G\hat{p}_0}{1 - G(\hat{p}_0 + \hat{p}_G)},$$

(5)

where $\hat{p}_G = \frac{n_G^{(2)}}{n_2}$, $\hat{p}_0 = \frac{n_0^{(2)}}{n_2}$ and assuming that $(\hat{p}_0 + \hat{p}_G) \neq \frac{1}{G}$. 

Ratio estimator

Biased estimator BUT asymptotically unbiased
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- Ratio estimator
- Biased estimator BUT asymptotically unbiased
The dominating term of its bias is given by the following result:

**Theorem**

For the estimator $\hat{\theta}$ given by (5), we have that

$$E(\hat{\theta} - \theta) \approx \frac{G(p_0 + p_G)[\theta G(1 - p_0 - p_G) + A - G(1 - p_0)]}{n_2[1 - G(p_0 + p_G)]^2},$$

where $A = \sum_{k=1}^{G} kp_k$.
Theorem

\[ Var(\hat{\theta}) \approx \frac{1}{[1 - G(p_0 + p_G)]^2} \times \left( \frac{Var(W_1)}{n_1} + \frac{Var(W_2)}{n_2} + \frac{G^2 p_0(1 - p_0) - 2Gp_0A}{n_2} \right), \]

where \( W_1 \) and \( W_2 \) are random variables with probability mass function

\[ P(W_1 = k) = q_k, \quad k = 1, \ldots, G \]

and

\[ P(W_2 = k) = p_k, \quad k = 0, 1, \ldots, G \]

respectively.
Obviously the approximate variance is unknown
Obviously the approximate variance is **unknown**

Thus, an estimator of the variance can be given as:
Obviously the approximate variance is **unknown**

Thus, an estimator of the variance can be given as:

\[
\hat{\text{Var}}(\hat{\theta}) = \frac{1}{[1 - G(\hat{p}_0 + \hat{p}_G)]^2} \times \left( \frac{S_1^2}{n_1} + \frac{S_2^2}{n_2} + G^2 \frac{\hat{p}_0 (1 - \hat{p}_0)}{n_2} - 2GC_2 \hat{p}_0 \frac{n_2}{n_2} \right),
\]

where \(S_1^2\) and \(S_2^2\) are the sample variances of the numbers reported by the first and second sample respectively.
3. Simulation Results

Assumptions:

\[ n_1 = 500, \quad n_2 = 800, \quad G = 5, \quad \theta = 0.25 \]

Population prevalences for the five non-sensitive items:

\[ 0.10, \quad 0.15, \quad 0.20, \quad 0.25, \quad 0.30 \] respectively

Table 1 provides some revealing results for the revised version and Table 2 provides some revealing results for the original ICT.
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- **Table 1** provides some revealing results for the revised version and **Table 2** provides some revealing results for the original ICT
### Table 1

<table>
<thead>
<tr>
<th>Point Estimate</th>
<th>Left Limit</th>
<th>Right Limit</th>
<th>Length of Conf.Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2232500</td>
<td>0.11923745</td>
<td>0.3272625</td>
<td>0.2080251</td>
</tr>
<tr>
<td>0.2945000</td>
<td>0.18995337</td>
<td>0.3990466</td>
<td>0.2090933</td>
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</tr>
<tr>
<td>0.2015000</td>
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<td>0.3050789</td>
<td>0.2071578</td>
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<tr>
<td>0.3090000</td>
<td>0.20162572</td>
<td>0.4163743</td>
<td>0.2147486</td>
</tr>
<tr>
<td>0.2166038</td>
<td>0.10961250</td>
<td>0.3235951</td>
<td>0.2139826</td>
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</table>

### Table 2

<table>
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</table>
According to Table 1 and Table 2 the mean value of the point estimates for $\theta$ is: 0.2549394 in the improved version and 0.2548125 in the classical ICT.
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Also, in the improved version, for the generated confidence intervals, the minimum is 0.1989, the maximum length is 0.2210 and the mean value of the length is 0.2111.
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The corresponding values for the classical ICT are: 0.1989, 0.2210 and 0.2109.
For the revised method Table 3 gives actual coverage proportions in the case of the repetition of the procedure one hundred times.
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The corresponding results for the classical ICT are given in **Table 5** and **Table 6**.
### Table 3

**Actual Coverage Proportions**

<table>
<thead>
<tr>
<th>0.94</th>
<th>0.97</th>
<th>0.91</th>
<th>0.96</th>
<th>0.94</th>
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### Table 4

<table>
<thead>
<tr>
<th>Min.</th>
<th>1st Qu.</th>
<th>Median</th>
<th>Mean</th>
<th>3rd Qu.</th>
<th>Max.</th>
</tr>
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<td>0.8800</td>
<td>0.9375</td>
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<tr>
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Table 6

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<th>Min.</th>
<th>1st Qu.</th>
<th>Median</th>
<th>Mean</th>
<th>3rd Qu.</th>
<th>Max.</th>
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<td>0.9450</td>
<td>0.9465</td>
<td>0.9700</td>
<td>0.9900</td>
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</tbody>
</table>
Finally, we generated the appropriate code in R in order to find the Mean Square Error (MSE) of the estimator $\hat{\theta}$ for the improved version.
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The results are presented in **Table 7**. The mean value of the MSE is 0.002954452.
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<table>
<thead>
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</table>
For the classical ICT the MSE is obviously equal with the variance of the estimator \( \theta \). The results are shown in Table 8.
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Taking the mean value of the generated results we find that the MSE is 0.002941434.
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<th>Table 8</th>
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<tbody>
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</table>
4. Discussion

- Equation (4), namely equation

\[ \theta = \frac{E(C_1 - C_2) - Gp_0}{1 - G(p_0 + p_G)} \]

is well defined if we assume that \( 1 - G(p_0 + p_G) \neq 0 \)

- The assumption will always be true if the list of non sensitive items contains two items for which the summation of their population prevalences is less than \( \frac{1}{G} \)
It is achievable to have two non sensitive items for which the summation of their population prevalence is less than $\frac{1}{G}$.

In this case the probability that all $G$ non sensitive items are applicable to an individual will also be less than $\frac{1}{G}$.

If the list of the non sensitive items includes two items with the summation of their population prevalences less than $\frac{1}{G}$, then it is expected that $(\hat{p}_0 + \hat{p}_G)$ will be less than $\frac{1}{G}$, so that equation (5) is well defined.

In case however that $1 - G(\hat{p}_0 + \hat{p}_G) = 0$, it is clear that equation (5) is not applicable.
Instead, equation (1) suggests that $\theta$ can be estimated from its sample equivalent,

$$\hat{q}_1 = \frac{n_1^{(1)}}{n_1} - \hat{p}_1 = \left(\frac{1}{G} - \hat{p}_1\right) \hat{\theta} + \hat{p}_1,$$

where $\hat{p}_1$ is the estimator of $p_1$ obtained from the second sample. Then

$$\hat{\theta} = \left(\frac{n_1^{(1)}}{n_1} - \hat{p}_1\right) \left(\frac{1}{G} - \hat{p}_1\right)^{-1},$$

which is well defined if $\hat{p}_1 \neq \frac{1}{G}$.
However if \((\hat{p}_0 + \hat{p}_G) = \hat{p}_1 = \frac{1}{G}\), then equation (2) for \(k = 2\) suggests that \(\theta\) can be estimated from:

\[
\hat{q}_2 = \frac{n_2^{(1)}}{n_1} = \hat{p}_2 (1 - \hat{\theta}) + \frac{1}{G} \hat{\theta},
\]

where \(\hat{p}_2\) is the estimator of \(p_2\) obtained again from the second sample. As previously, from the above calculations we get that:

\[
\hat{\theta} = \left( \frac{n_2^{(1)}}{n_1} - \hat{p}_2 \right) \left( \frac{1}{G} - \hat{p}_2 \right)^{-1},
\]

which is well defined if \(\hat{p}_2 \neq \frac{1}{G}\).
If \((\hat{p}_0 + \hat{p}_G) = \hat{p}_1 = \hat{p}_2 = \frac{1}{G}\), then if we continue in the same manner we will get an estimator by using the sample equivalent of equation (2) for some \(k\).

The entire process will only fail if

\[(\hat{p}_0 + \hat{p}_G) = \hat{p}_1 = \hat{p}_2 = \ldots = \hat{p}_{G-1} = \frac{1}{G}\]

But this scenario is unreasonable and it is not to be expected!
Concluding Remarks

Practically, there is not any potential difference between the MSE of the 2 methods. The new version presented offers better protection than the original ICT. The new version does not give unbiased estimators, but the main point is to get accurate results.
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Thank you very much for your attention!