ON SOME PROPERTIES OF MORTALITY RATES

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SUMMARY

The first part of the paper underlines the necessity to consider in the analysis of mortality the double nature of general age-specific mortality rates: they determine with the number and age distribution of persons exposed to the risk of dying the number and the age distribution of the deceased. An attempt is made to separate the impact of these two roles.

The second part of the contribution describes the method of decomposition of the differences between the life expectancies at birth (and at higher ages) elaborated and used in the Demographic Research Institute of the HCSO, based on the evidence that the life expectancy at birth may be defined, among others, as the mean age of all the deceased of the life table and this mean age is equal to the weighted arithmetic mean of the mean ages of victims of different causes of death.

KEYWORDS: Mortality rates; Life expectancies; Causes of death.

Changing age-specific mortality rates always lead to the change of all the other life-table functions. The intensity of the phenomenon studied (i.e. mortality) remains equal to unity in all cases and the distribution of the deceased of the life table by ages changes in all cases. Life expectancy at birth remains equal among others to the mean age of the deceased of the life-table in all cases and this mean age remains equal to the weighted mean of the mean ages of victims of different causes of death in all cases. The decomposition of the differences between the two life expectancies is therefore the decomposition of the differences between the two weighted arithmetic means in all cases.

Several methods of decomposing the differences between the life-expectancies at birth have already been elaborated and published. The general age-specific mortality rates and the age- and cause-specific mortality rates have a certain role in all of them, but solely or almost solely in the distribution of the gains (or losses) in the number of person-years by causes of death studied. Their influence on the number and distribution by age and causes of death of the deceased of the life-tables compared is entirely neglected in all of them.

The method elaborated and used for this purpose in the Demographic Research Institute of the HCSO starts from distributing the deceased in the death function of the life-

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table compared by causes of death. We are therefore highly interested in studying the already known and unknown or simply neglected properties of general age-specific mortality rates and of age- and cause-specific mortality rates influencing the distribution of the deceased of the life table by age and causes of death studied.

1. The double nature of the general age-specific and age- and cause-specific mortality rates

The general age-specific mortality rates with the number and age distribution of persons exposed to the risk of dying, immediately determine the number and the age distribution of the deceased. They have therefore a double nature. If we consider an age interval with a given number of those exposed to the risk of dying, a higher value of the corres-ponding age-specific mortality rate produces a higher number and a lower value a lower number of the deceased. If the number of those exposed to the risk of dying is given for all the age groups, it is easy to establish which from the two series of age-specific mortality rates produces a higher or a lower number of the deceased. In a separate age group a higher rate produces more and a lower rate produces less of them. This is not true if we consider the sum of general age-specific mortality rates. A higher sum may produce the same or a lower and a lower sum the same or a higher total number of deceased persons because the number of the deceased does not only depend on the level of the rates, but it also depends on some other still neglected properties of them. It is obviously true that if in one of the series of the age-specific mortality rates all the values are lower than in the other one, the number of the deceased and the number of years they lived in different age groups and the total number of the deceased and of years they lived will be lower the and inversely. Nevertheless it may happen that the lower the values of all the rates, and the lower the value of their sum, then a lower number of deceased and a lower number of years they lived in all the age groups is connected with a higher number of years per one deceased (the total number of years lived divided by the total number of deceased). Such a situation is presented in Table 1.

Column (1) of Table 1 shows the age groups, column (2) the mean ages at death in different age groups (calculated by using an appropriate weighting procedure), column (3) the number of those exposed to the risk of dying in different age groups (equal in this case to the number of years in different age groups ($n^{(M)} = n^{(F)}$), columns (4) and (5) the general age-specific mortality rates of Hungarian males and females in 1966, columns (6) and (7) the number of deceased males and females. Column (8) shows that the number of deceased males is higher in all the age-groups, columns (9) and (10) present the number of years lived by the deceased males and females. It is clear that the total number of years lived by deceased females is lower than that lived by deceased males, nevertheless the total number of years divided by the total number of the deceased is higher in the case of females (84.200390 > 83.077122). This fact may only be explained by an until now neglected property of the series of general age-specific mortality rates: that is the ratios of the values of neighbouring rates in these series are different. The values of the rates experienced after childhood at higher ages exceed much more the rates experienced at younger ages by females. More precisely: their descent during the years of early childhood and their ascent after the minimum value attained is quicker than in the case of males.

Table 1

The double role of general age-specific mortality rates using the data of the Hungarian male and female population for 1966

Age groups (years) $x, x+n$	$\bar{x}$	$n^{(M)} = n^{(F)}$	${}_nm_x^{(M)}$	${}_nm_x^{(F)}$	$n_nm_x^{(M)}$	$n_nm_x^{(F)}$	$n_nm_x^{(F)} - n_nm_x^{(M)}$	$\bar{x}\,n_nm_x^{(M)}$	$\bar{x}\,n_nm_x^{(F)}$	$\bar{x}(n^{(F)}-n^{(M)})\,{}_nm_x^{(M)}$	$\bar{x}(n_nm_x^{(F)}-n_nm_x^{(M)})$	(11)+(12)
(1)	(2)	(3)	(4)	(5)	(6)=(3)·(4)	(7)=(3)·(5)	(8)=(7)-(6)	(9)=(12)·(6)	(10)=(2)·(7)	(11)	(12)=(2)·(8)	(13)
0	0.13935	1	0.045125	0.036906	0.045125	0.036906	-0.008219	0.006288	0.005143	0	-0.001145	-0.001145
1–4	2.50210	4	0.001235	0.001009	0.004940	0.004036	-0.000904	0.012360	0.010098	0	-0.002262	-0.002262
5–9	7.49737	5	0.000448	0.000269	0.002240	0.001345	-0.000895	0.016794	0.010084	0	-0.006710	-0.006710
10–14	12.63333	5	0.000438	0.000255	0.002190	0.001275	-0.000915	0.027667	0.016107	0	-0.011560	-0.011560
15–19	18.10476	5	0.000913	0.000502	0.004565	0.002510	-0.002055	0.082648	0.045443	0	-0.037205	-0.037205
20–24	22.87185	5	0.001373	0.000560	0.006865	0.002800	-0.004065	0.157015	0.064041	0	-0.092974	-0.092974
25–29	27.78945	5	0.001436	0.000723	0.007180	0.003615	-0.003565	0.199528	0.100459	0	-0.099069	-0.099069
30–34	33.10997	5	0.001819	0.000909	0.009095	0.004545	-0.004550	0.301135	0.150485	0	-0.150650	-0.150650
35–39	37.75633	5	0.002551	0.001495	0.012755	0.007475	-0.005280	0.481582	0.282229	0	-0.199353	-0.199353
40–44	42.80283	5	0.003399	0.002314	0.016995	0.011570	-0.005425	0.727434	0.495229	0	-0.232205	-0.232205
45–49	47.69929	5	0.004844	0.003319	0.024220	0.016595	-0.007625	1.155277	0.791570	0	-0.363707	-0.363707
50–54	52.68968	5	0.008590	0.005305	0.042950	0.026525	-0.016425	2.263022	1.397594	0	-0.865428	-0.865428
55–59	57.67475	5	0.013779	0.008137	0.068895	0.040685	-0.028210	3.973502	2.346497	0	-1.627005	-1.627005
60–64	62.65640	5	0.023267	0.013425	0.116335	0.067125	-0.049210	7.289132	4.205811	0	-3.083321	-3.083321
65–69	67.63221	5	0.037467	0.023770	0.187335	0.118850	-0.068485	12.669880	8.038088	0	-4.631792	-4.631792
70–74	72.59156	5	0.058186	0.042243	0.290930	0.211215	-0.079715	21.119063	15.332426	0	-5.786637	-5.786637
75–79	77.52282	5	0.092015	0.073663	0.460075	0.368315	-0.091760	35.666311	28.552817	0	-7.113494	-7.113494
80–84	82.40913	5	0.146590	0.125405	0.732950	0.627025	-0.105925	60.401772	51.672585	0	-8.729187	-8.729187
85–	89.44615	15	0.236192	0.218551	3.542880	3.278265	-0.264615	316.896976	293.228183	0	-23.668793	-23.668793
Total	–	100	–	–	5.578520	4.830677	-0.747843	463.447386	406.744889	0	-56.702497	-56.702497
Average	–	–	–	–	–	–	–	83.077122	84.200390	0	1.123268	1.123268

Source: Here and in the following tables the data of the Hungarian male and female population for 1966 are used.

Table 2

The influence of general age-specific mortality rates on the change of the age structure of the deceased

Age groups (years) x, $x+n$	$\bar{x}$	$n^{(M)} = n^{(F)}$	${}_n m_x^{(M)}$	${}_n m_x^{(F)} \left(\frac{\sum n_n m_x^{(M)}}{\sum n_n m_x^{(F)}} \right)$	$n_n m_x^{(M)}$	$n_n m_x^{(F)} \left(\frac{\sum n_n m_x^{(M)}}{\sum n_n m_x^{(F)}} \right)$	$n_n m_x^{(F)} \left(\frac{\sum n_n m_x^{(M)}}{\sum n_n m_x^{(F)}} \right) - n_n m_x^{(M)}$	$\bar{x} n_n m_x^{(M)}$	$\bar{x} n_n m_x^{(F)} \left(\frac{\sum n_n m_x^{(M)}}{\sum n_n m_x^{(F)}} \right)$	$\bar{x} (n^{(F)} - n^{(M)}) n_n m_x^{(M)}$	$\bar{x} \left[n_n m_x^{(F)} \left(\frac{\sum n_n m_x^{(M)}}{\sum n_n m_x^{(F)}} \right) - n_n m_x^{(M)} \right]$	(11)+(12)
(1)	(2)	(3)	(4)	(5)	(6)=(3)·(4)	(7)=(3)·(5)	(8)=(7)-(6)	(9)=(12)·(6)	(10)=(2)·(7)	(11)	(12)=(2)·(8)	(13)
0	0.13935	1	0.045125	0.042620	0.045125	0.042620	-0.002505	0.006288	0.005939	0	-0.000349	-0.000349
1-4	2.50210	4	0.001235	0.001165	0.004940	0.004660	-0.000279	0.012360	0.011662	0	-0.000698	-0.000698
5-9	7.49737	5	0.000448	0.000311	0.002240	0.001555	-0.000687	0.016794	0.011643	0	-0.005151	-0.005151
10-14	12.63333	5	0.000438	0.000294	0.002190	0.001470	-0.000718	0.027667	0.018596	0	-0.009071	-0.009071
15-19	18.10476	5	0.000913	0.000580	0.004565	0.002900	-0.001666	0.082648	0.052486	0	-0.030162	-0.030162
20-24	22.87185	5	0.001373	0.000647	0.006865	0.003235	-0.003631	0.157015	0.073968	0	-0.083047	-0.083047
25-29	27.78945	5	0.001436	0.000835	0.007180	0.004175	-0.003005	0.199528	0.116021	0	-0.083507	-0.083507
30-34	33.10997	5	0.001819	0.001050	0.009095	0.005250	-0.003846	0.301135	0.173794	0	-0.127341	-0.127341
35-39	37.75633	5	0.002551	0.001726	0.012755	0.008630	-0.004123	0.481582	0.325913	0	-0.155669	-0.155669
40-44	42.80283	5	0.003399	0.002672	0.016995	0.013360	-0.003634	0.727434	0.571889	0	-0.155545	-0.155545
45-49	47.69929	5	0.004844	0.003833	0.024220	0.019165	-0.005056	1.155277	0.914109	0	-0.241168	-0.241168
50-54	52.68968	5	0.008590	0.006126	0.042950	0.030630	-0.012319	2.263022	1.613938	0	-0.649084	-0.649084
55-59	57.67475	5	0.013779	0.009397	0.068895	0.046985	-0.021912	3.973502	2.709733	0	-1.263769	-1.263769
60-64	62.65640	5	0.023267	0.015503	0.116335	0.077515	-0.038818	7.289132	4.856936	0	-2.432196	-2.432196
65-69	67.63221	5	0.037467	0.027450	0.187335	0.137250	-0.050086	12.669880	9.282453	0	-3.387427	-3.387427
70-74	72.59156	5	0.058186	0.048783	0.290930	0.243915	-0.047017	21.119063	17.706025	0	-3.413037	-3.413037
75-79	77.52282	5	0.092015	0.085067	0.460075	0.425335	-0.034741	35.666311	32.973091	0	-2.693219	-2.693219
80-84	82.40913	5	0.146590	0.144819	0.732950	0.724095	-0.008854	60.401772	59.672121	0	-0.729651	-0.729651
85-	89.44615	15	0.236192	0.252385	3.542880	3.785775	0.242897	316.896976	338.623177	0	21.726201	21.726201
Total	–	100	0.679667	0.645263	5.578520	5.578520	0.000000	463.447386	469.719494	0	6.266109	6.266109
Average	–	–	–	–	–	–	–	83.077122	83.200378	0	1.123256	1.123256

Table 3

Sex differences, sex ratios, magnitudes related to the lowest values and ratios between neighbouring values of general age-specific mortality rates

Age groups (years) $x, x+n$	${}_n m_x^{(M)}$	${}_n m_x^{(F)}$	${}_n m_x^{(M)} - {}_n m_x^{(F)}$	${}_n m_x^{(M)} / {}_n m_x^{(F)}$	$\frac{{}_n m_x^{(M)}}{{}_5 m_{10}^{(M)}} x100$	$\frac{{}_n m_x^{(F)}}{{}_5 m_{10}^{(F)}} x100$	(6)-(7)	(6)/(7)	$\frac{{}_n m_{x+n}^{(M)}}{{}_n m_x^{(M)}}$	$\frac{{}_n m_{x+n}^{(F)}}{{}_n m_x^{(F)}}$	(10)-(11)	(10)/(11)
(1)	(2)	(3)	(4)=(2)-(3)	(5)=(2/3)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
0	0.045125	0.036906	0.008219	1.222701	10 303	14 473	-4 170	0.711846	0.027368	0.027340	0.000029	1.001050
1–4	0.001235	0.001009	0.000226	1.223984	282	396	-114	0.712594	0.362753	0.266601	0.096152	1.360659
5–9	0.000448	0.000269	0.000179	1.665428	102	105	-3	0.969598	0.977679	0.947955	0.029723	1.031355
10–14	0.000438	0.000255	0.000183	1.717647	100	100	0	1.000000	2.084475	1.968627	0.115847	1.058847
15–19	0.000913	0.000502	0.000411	1.818725	208	197	12	1.058847	1.503834	1.115538	0.388296	1.348079
20–24	0.001373	0.000560	0.000813	2.451786	313	220	94	1.427409	1.045885	1.291071	-0.245187	0.810091
25–29	0.001436	0.000723	0.000713	1.986169	328	284	44	1.156331	1.266713	1.257261	0.009452	1.007518
30–34	0.001819	0.000909	0.000910	2.001100	415	356	59	1.165024	1.402419	1.644664	-0.242246	0.852708
35–39	0.002551	0.001495	0.001056	1.706355	582	586	-4	0.993426	1.332419	1.547826	-0.215407	0.860832
40–44	0.003399	0.002314	0.001085	1.468885	776	907	-131	0.855173	1.425125	1.434313	-0.009188	0.993594
45–49	0.004844	0.003319	0.001525	1.459476	1 106	1 302	-196	0.849695	1.773328	1.598373	0.174955	1.109458
50–54	0.008590	0.005305	0.003285	1.619227	1 961	2 080	-119	0.942701	1.604075	1.533836	0.070239	1.045793
55–59	0.013779	0.008137	0.005642	1.693376	3 146	3 191	-45	0.985870	1.688584	1.649871	0.038713	1.023464
60–64	0.023267	0.013425	0.009842	1.733110	5 312	5 265	47	1.009002	1.610306	1.770577	-0.160271	0.909481
65–69	0.037467	0.023770	0.013697	1.576231	8 554	9 322	-767	0.917668	1.552993	1.777156	-0.224163	0.873864
70–74	0.058186	0.042243	0.015943	1.377412	13 284	16 566	-3 281	0.801918	1.581394	1.743792	-0.162398	0.906871
75–79	0.092015	0.073663	0.018352	1.249135	21 008	28 887	-7 879	0.727236	1.593110	1.702415	-0.109305	0.935794
80–84	0.146590	0.125405	0.021185	1.168933	33 468	49 178	-15 710	0.680543	1.611242	1.742761	-0.131519	0.924534
85–	0.236192	0.218551	0.017641	1.080718	53 925	85 706	-31 781	0.629185	–	–	–	–

Table 4

The difference quotients and curvatures of the empirical curves of general age-specific mortality rates

Age groups (years) x, $x+n$	Approximate values of difference quotients		(2)-(3)	(2)/(3)	The angles calculated from difference quotients (DEG)		The differences between the neighbouring angles		The mean arc length		The mean curvatures	
	in the case of males	in the case of females			in the case of males	in the case of females	in the case of males	in the case of females	in the case of males	in the case of females	in the case of males	in the case of females
(1)	(2)	(3)	(4)	(5)	(6)=arc tan (2)	(7)=arc tan (3)	(8)	(9)	(10)	(11)	(12)=(8)/(10)	(13)=(9)/(11)
0	-18.576	-15.193	-3.383	1.223	93.0814	93.7658	77.9401	77.8155	43.954	35.975	1.773	2.163
1–4	-0.158	-0.148	-0.010	1.068	171.0215	171.5813	8.8639	8.2468	5.057	5.050	1.753	1.633
5–9	-0.002	-0.003	0.001	0.667	179.8854	179.8281	-174.9132	-177.2515	5.136	5.136	-34.056	-34.512
10–14	0.087	0.045	0.042	1.933	4.9722	2.5766	0.5114	-1.8891	5.492	5.477	0.093	-0.345
15–19	0.096	0.012	0.084	8.000	5.4836	0.6875	-4.7388	1.2026	4.789	4.767	-0.990	0.252
20–24	0.013	0.033	-0.020	0.394	0.7448	1.8901	3.3734	0.1145	4.918	4.920	0.686	0.023
25–29	0.072	0.035	0.037	2.057	4.1182	2.0045	4.8603	5.1769	5.334	5.324	0.911	0.972
30–34	0.158	0.126	0.032	1.254	8.9785	7.1814	0.5581	2.0205	4.704	4.683	0.119	0.431
35–39	0.168	0.162	0.006	1.037	9.5366	9.2020	6.8994	2.1630	5.117	5.113	1.348	0.423
40–44	0.295	0.201	0.094	1.468	16.4361	11.3650	20.4705	10.3375	5.105	5.091	4.010	2.031
45–49	0.751	0.398	0.353	1.887	36.9065	21.7026	9.2443	7.8940	6.240	5.371	1.481	1.470
50–54	1.041	0.568	0.473	1.833	46.1508	29.5966	16.1527	17.0987	7.196	5.733	2.245	2.983
55–59	1.905	1.061	0.844	1.795	62.3035	46.6953	8.3868	17.6171	10.716	7.265	0.783	2.425
60–64	2.854	2.079	0.775	1.373	70.6903	64.3124	5.8493	10.6605	15.047	11.479	0.389	0.929
65–69	4.178	3.725	0.453	1.122	76.5395	74.9729	5.1667	6.1081	21.304	19.127	0.243	0.319
70–74	6.860	6.372	0.488	1.077	81.7063	81.0809	3.1775	3.5242	34.187	31.805	0.093	0.111
75–79	11.169	10.589	0.580	1.055	84.8837	84.6051	0.6257	1.0746	54.793	51.972	0.011	0.021
85–	12.733	13.237	-0.504	0.962	85.5094	85.6797	–	–	89.878	93.411	–	–

If we multiply the series of general age-specific mortality rates of females by a constant that makes them produce as many deceased females as the number of deceased males (5.578520/4.830677 = 1.154811), the number of deceased females in the oldest age group will already be higher than that of the males, but the total number of years lived by deceased females divided by the number of deceased females will differ from the value of this indicator just like in the previous situation. (See Table 2.) If we use another multiplier (e.g. 0.5) the result will be the same. It is clear that the series of general age-specific mortality rates for females produce, ceteris paribus, a higher mean age of the deceased. The age-structure of the sum of these rates is also different: it is older in the case of females and younger in the case of males.

In the past a considerable number of authors along with the United Nations Secretariat considered only the differences between the corresponding elements of general age-specific mortality rates e.g. $({}_n m_x^{(M)} - {}_n m_x^{(F)})$, the sex differences of rates and their ratios e.g. $({}_n m_x^{(M)} / {}_n m_x^{(F)})$, the sex ratios of rates. In our case they are shown in columns (4) and (5) in Table 3. The following columns of Table 3 already show the properties of general age-specific mortality rates which have been neglected up to now.

Columns (6) and (7) in Table 3 show that if we divide all the rates by the lowest rate in both series, i.e. the rate for 10–14 years of age and multiply the results of the division by 100, the rates for females obtained this way will be higher at younger ages and mainly at higher ages than the rates for males despite the fact that in reality the general age-specific mortality rates in all the age groups are lower in the case of females (See Figure 1.) The differences and ratios of these artificially created figures rise in both directions from the age-group 10–14.

Figure 1. The general age-specific mortality rates for Hungarian males and females related to their lowest values in the age group 10–14, 1966

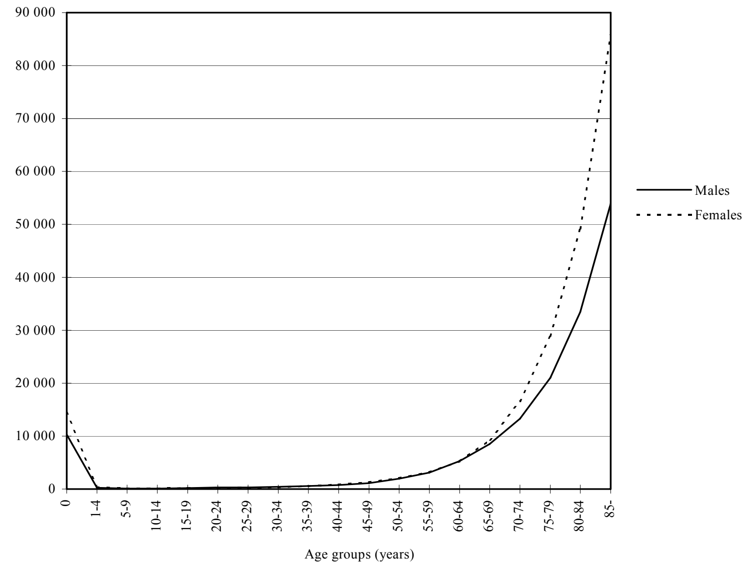


Table 5

Sums, cumulated values, the structure of the sums and cumulated values, the distance of the cumulated values from the sums and the multipliers which assure that the sums reach the general age-specific mortality rates

Age groups (years) x $x+n$	$n_n m_x^{(M)}$	$n_n m_x^{(F)}$	The percentage distribution of the sums of the values of		The cumulated values of the		The cumulated values of the		The distance of the cumulated values of		The multipliers of the cumulated values of	
			$n_n m_x^{(M)}$	$n_n m_x^{(F)}$	$n_n m_x^{(M)}$	$n_n m_x^{(F)}$	$n_n m_x^{(M)}$	$n_n m_x^{(F)}$	$n_n m_x^{(M)}$	$n_n m_x^{(F)}$	$n_n m_x^{(M)}$	$n_n m_x^{(F)}$
							calculated form the percentage distributions		from their sums		assuring the achievement of their sums	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
0	0.045125	0.036906	0.808906	0.763992	–	–	–	–	1.000000	1.000000	–	–
1–4	0.004940	0.004036	0.088554	0.083549	0.045125	0.036906	0.808906	0.763992	0.991911	0.992360	122.623712	129.891373
5–9	0.002240	0.001345	0.040154	0.027843	0.050065	0.040942	0.897460	0.847542	0.991025	0.991525	110.425547	116.988301
10–14	0.002190	0.001275	0.039258	0.026394	0.052305	0.042287	0.937614	0.875385	0.990624	0.991246	105.653666	113.235510
15–19	0.004565	0.002510	0.081832	0.051960	0.054495	0.043562	0.976872	0.901778	0.990231	0.990982	101.367557	109.891993
20–24	0.006865	0.002800	0.123061	0.057963	0.059060	0.046072	1.058704	0.953738	0.989413	0.990463	93.455130	103.850603
25–29	0.007180	0.003615	0.128708	0.074834	0.065925	0.048872	1.181765	1.011701	0.988182	0.989883	83.619188	97.843448
30–34	0.009095	0.004545	0.163036	0.094086	0.073105	0.052487	1.310473	1.086535	0.986895	0.989135	75.308324	91.035685
35–39	0.012755	0.007475	0.228645	0.154740	0.082200	0.057032	1.473509	1.180621	0.985265	0.988194	66.865207	83.701168
40–44	0.016995	0.011570	0.304651	0.239511	0.094955	0.064507	1.702154	1.335361	0.982978	0.986646	57.749092	73.886090
45–49	0.024220	0.016595	0.434165	0.343534	0.111950	0.076077	2.006805	1.574872	0.979932	0.984251	48.830460	62.497207
50–54	0.042950	0.026525	0.769917	0.549095	0.136170	0.092672	2.440970	1.918406	0.975590	0.980816	39.967320	51.126608
55–59	0.068895	0.040685	1.235005	0.842221	0.179120	0.119197	3.210887	2.467501	0.967891	0.975325	30.144038	39.526834
60–64	0.116335	0.067125	2.085410	1.389557	0.248015	0.159882	4.445892	3.309722	0.955541	0.966903	21.492672	29.214014
65–69	0.187335	0.118850	3.358149	2.460318	0.364350	0.227007	6.531302	4.699279	0.934687	0.953007	14.310882	20.279859
70–74	0.290930	0.211215	5.215183	4.372369	0.551685	0.345857	9.889451	7.159597	0.901105	0.928404	9.111785	12.967267
75–79	0.460075	0.368315	8.247259	7.624501	0.842615	0.557072	15.104633	11.531965	0.848954	0.884680	5.620485	7.671549
80–84	0.732950	0.627025	13.138789	12.980065	1.302690	0.925387	23.351893	19.156466	0.766481	0.808435	3.282308	4.220170
85–	3.542880	3.278265	63.509318	67.863469	2.035640	1.552412	36.490682	32.136531	0.635093	0.678635	1.740426	2.111724
Total	5.578520	4.830677	100.000000	100.000000	5.578520	4.830677	100.000000	100.000000	0.000000	0.000000	0.000000	0.000000

Columns (10) and (11) in Table 3 show the ratios of neighbouring rates separately in both series $({}_n m_{x+n}^{(M)} / {}_n m_x^{(M)})$ and $({}_n m_{x+n}^{(F)} / {}_n m_x^{(F)})$. The conclusions remain the same as before.

Columns (2) and (3) in Table 4 show the so called difference quotiens (i.e. the differences between the ordinate values divided by the differences between the abscissa values) calculated separately in both series. If we had the possibility to work with continuous and differentiable functions of age-specific mortality rates, it would be possible to calculate the differential coefficients or derivatives with respect to *x* (the age) so as to work with tangents at different points to the curves instead of secants. Nevertheless it is possible even when working with secants, – i.e. straight lines joining two points of the curves – to calculate the slopes and the differences between the tangents and angles made by these lines with the age axis and calculate the curvatures – i.e. the rates of the changes of the angles between the tangents with respect to the different arcs of the curves – and show that the curvature is e.g. at older age higher in the case of females than in the case of males. We may easily separate the monotonically descending and ascending segments of the empirical curves and distinguish the parts of the curves which are concave downwards and concave upwards even in our case.

Even in our case we would need a good approximation the length of the arc of the empirical curves. It is possible to demonstrate that in the case of females we obtain a curve, in the oldest age group, the arc of which is linked with a higher mean age of all the deceased of the life-table in question. This, however, does not mean a higher total length of life.

Columns (2) and (3) in Table 5 contain the values of the general age-specific mortality rates multiplied by the length of the age groups. The age-specific probabilities of surviving and dying of corresponding life tables may immediately be calculated by using the simple exponential formula ${}_n p_x = exp(-n_n m_x)$ and ${}_n q_x = 1 - {}_n p_x = 1 - exp(-n_n m_x)$, or the formulae of *Reed* and *Merrell*, of *Greville*, of *Keyfitz* and *Frauenthal*, etc. The same is true for the probabilities of surviving and dying from the exact age 0 to the exact age *x*, i.e. practically all life-table functions may already be calculated by using them. The sum of the multiplied general age-specific mortality rates is smaller in the case of females than in the case of males.

Columns (4) and (5) of Table 5 and Figure 2 show the age-structure (the percentage distribution) of the sums of these two series of multiplied rates. The elements of this distribution for younger ages are smaller and for older ages higher in the case of females.

Columns (6) and (7) present the cumulated values of these two series of multiplied rates. Columns (8) and (9) show the same calculated by using the data of their percentage distribution (included in columns (4) and (5) (see Figure 3). The figures in column (9) are smaller than those in column (8) and in the case of the female population they approach the upper limit 100 percent slower than in the case of the males. This slow convergence is also linked with a lower mortality level, i.e. with a longer life expectancy at birth of females.

Columns (10) and (11) in Table 5 show the distance between the cumulated general age-specific mortality rates and their sums. If we denote this distance by v_a, we may calculate it by using the formula $v_a = \left(\sum_{x=0}^{\omega} n_n m_x - \sum_{x=0}^{x=a} n_n m_x \right) / \sum_{x=0}^{\omega} n_n m_x$. The data in columns (6) and (7), and in columns (8) and (9) are both appropriate for realising this calculation. The cumulated values of the general age-specific mortality rates may be reproduced by using the *Baule–Mitscherlich* saturation function: $\sum_{x=0}^{x=a} {}_n m_x = \sum_{x=0}^{\omega} {}_n m_x (1 - v_a)$, or $100(1 - v_a)$, where v_a denotes the distance in question. The values of the v_a are bigger at all ages in the case of females than in the case of males, which is also due to the lower mortality level, i.e. to the higher life expectancy of females at birth. In the case of the male population the values of v_{50} = (5.578520 - 0.136170)/5.578520 = (100 - 2.440970)/100 = 0.975590, and the value of multiplied rates cumulated from the age 0 to 50 = 5.578520 (1 - 0.975590), or in percentages 100(1 - 0.975590) = 2.440970.

Figure 2. Distribution by age groups of the sum of general age-specific mortality rates for Hungarian males and females, 1966

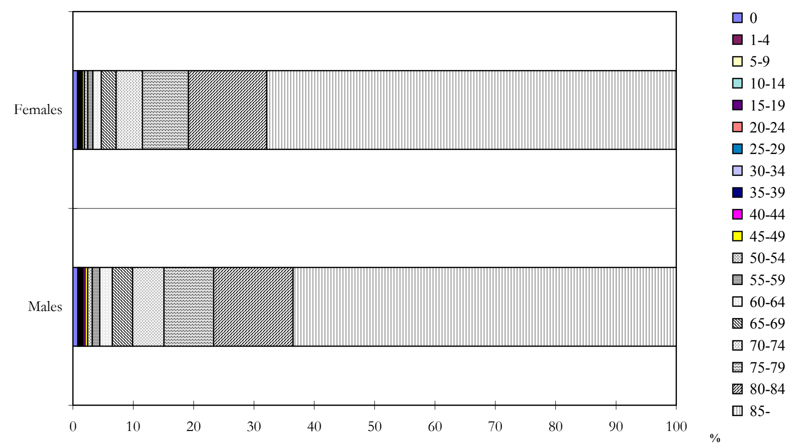
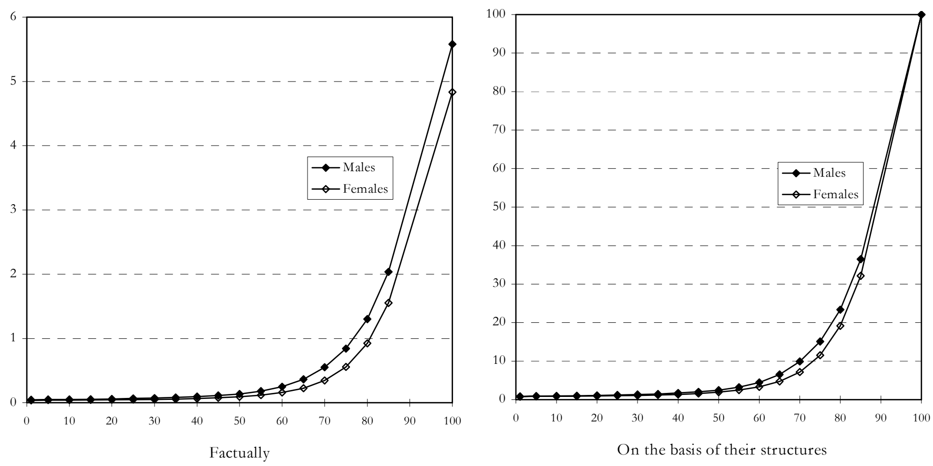


Figure 3. Cumulated values of general age-specific mortality rates of Hungarian males and females factually and on the basis of the age structures of their sums, 1966



Columns (12) and (13) in Table 5 show those multipliers of the cumulated values of the multiplied age-specific mortality rates $n_n m_x^{(F)}$ and $n_n m_x^{(N)}$ which assure that they reach their total sum. If we denote these multipliers by s_a, the formula used for their calculation may be written as follows: $s_a = \left(\sum_{x=0}^{\omega} n_n m_x - \sum_{x=0}^{x=a} n_n m_x \right) / \sum_{x=0}^{x=a} n_n m_x$. Their values may be calculated by using the data of columns (6) and (7) or columns (8) and (9). In the case of the male population the value of

$$s_{50} = (5.578520 — 0.136170)/0.136170 = (100 — 2.440970)/2.440970,$$

because

$$5.578520 — 0.136170 = 39.967320 \cdot 0.136170 = 5.442350$$

and

$$5.442350/39.967320 = 0.136170,$$

and if we use the data of the percentage distribution:

$$100 — 2.440970 = 39.967320 \cdot 2.440970 = 97.559030$$

and

$$97.559030/39.967320 = 2.440970.$$

The calculated values of general age-specific mortality rates may be reproduced by using the well-known logistic function

$$\sum_{x=0}^{x=a} n_n m_x = \sum_{x=0}^{\omega} n_n m_x \left(\frac{1}{1+s_a} \right) = \sum_{x=0}^{\omega} n_n m_x (1+s_a)^{-1} \quad \text{or} \quad 100 \left(\frac{1}{1+s_a} \right) = 100(1+s_a)^{-1}.$$

The sum of multiplied rates cumulated from age 0 to 50 equals $5.578850\,(1/(1+39.967320)) = 5.578850(1+39.967320)^{-1} = 0.136170$, or expressed in percentages $100\,(1/(1+39.967320)) = 100\,(1+39.967320)^{-1} = 2.440970$. The values of this multipliers s_a are higher in the case of the female population than in the case of the male population. This property of the general age-specific mortality rates is also linked with the lower mortality level, i.e. higher life expectancy at birth of females.

On the basis of the example we have just analysed, it is possible to state that when comparing two series of general age-specific mortality rates (and age- and cause-specific mortality rates) it is not sufficient to consider only their differences and their ratios, but, especially if we want to understand their roles in creating differences in mortality levels, it is necessary to consider the differences in their rates of descent and ascent, in their relative magnitudes, in their concavities, curvatures, difference quotients (or derivatives, if possible), the differences in the age structure of their sums, in the speed and acceleration of the convergence of their cumulated values to their sums, as well as the distances of their cumulated values from their sums, the differences of the multipliers which – in the different age groups – assure that they reach their sums by their cumulated values. These neglected properties of general age-specific mortality rates are related to their properties – which have already been considered many times –, i.e. to their differences and their ratios. We may formulate the following hypothesis: bigger differences between the corresponding values of the rates in question and their higher ratios involve bigger differences in their up until now neglected properties as well.

The same is true of the age- and cause-specific mortality rates with a few exceptions concerning mainly their absence in a few cases at some ages, the nature of their concavity and curvature, etc. which must be analysed in the case of each cause of death separately. It is very important to understand that they also have a double nature as well: they determine, with the number and age structure of those exposed to the risk of dying, the number and the age structure of the victims of given causes of death. In the case of the life tables by causes of death, the sum of the victims of different causes of death is equal to the radix of the life table $\left(l_0 = \sum_x \sum_i {}_n d_{i,x} \right)$ and thus it is easy to calculate the structure of the deceased in the life table by causes of death and the mean ages of victims of different causes of death. The mean age of all the deceased, as it has already been mentioned, is equal to the weighted arithmetic mean of the mean ages of victims of different causes of death.

In case of the period life-tables by causes of death, we may ask whether the number and age structure of those exposed to the risk of dying are really separate immediate determinants of the number and age structure of the deceased or are also determined by general age-specific mortality rates which are the sums of cause- and age-specific mortality rates. Demographers know that one of the possibilities of calculating the probabilities of surviving from birth to age a, if $l_0 = 1$, is the use of cumulated (or integrated) values of general age-specific mortality rates $l_a / l_0 = {}_a p_0 = exp\left(- \sum_{x=0}^{a} {}_n m_x \right)$ and $l_a/l_0 = {}_a p_0 = {}_a p_0^{(1)} \cdot {}_a p_0^{(2)} \cdot {}_a p_0^{(3)} \ldots$, where ${}_a p_0^{(1)}$, ${}_a p_0^{(2)}$, ${}_a p_0^{(3)}$ denote the corresponding probabilities of surviving from birth to the age a by causes of death denoted here by (1), (2), (3) etc. The number and age structure of exposed to risk of dying may easily be calculated by using simply the general age-specific or age- and cause-specific mortality rates and the total number of exposed to risk of dying in a period life table. If $l_0 = 1$, it is equal to the life expectancy at birth i.e. to the mean age of all the deceased in the life table

$$e_0^0 = \sum_{x=0}^{\omega} {}_n L_x = T_0 = \sum_{x=0}^{\omega} x d_x / \sum_{x=0}^{\omega} d_x \; .$$

Another important property of the general age-specific and age- and cause-specific mortality rates therefore, is that their already known and up until now neglected properties determine the number and age composition of the deceased by specifying the number and age composition of those exposed to the risk of dying as well: a higher mean age of all the deceased in a period life-table is, among others, the result of a higher mean age of those exposed to the risk of dying and inversely: a lower mean age of all the deceased is, among others, the result of a younger age structure of those exposed to the risk of dying.

Let us consider after this introduction the method of decomposing the differences between the life expectancies at birth elaborated and used in the Demographic Research Institute of the HCSO.

Table 6

The mean ages at death and the proportions of victimes of different causes of death according to the abridged Hungarian life-tables by causes of death for 1966

Causes of death studied	Infectious and parasitic diseases	Neoplasms (all forms)	Diseases of the circulatory system	Cerebrovas-cular diseases	Diseases of the respiratory systems	Diseases of the digestive system	Chronic liver disease and cirrhosis of liver	Congenital anomalies	Certain conditions originating in the perinatal period	Injury and poisoning	All other causes of death	Averages
	001–139	140–239	390–429 440–459	430–438	460–519	520–570 572–579	571	740–759	760–779	800–999		
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
Male population												
Mean ages at death (years)	63.495	68.272	74.375	75.658	65.450	63.034	64.535	1.051	0.145	52.674	68.733	67.528
Proportion of deceased (%)	3.266	19.323	35.325	14.987	4.933	2.918	1.036	0.739	2.781	7.452	7.241	100.000
Female population												
Mean ages at death (years)	63.141	68.497	78.388	77.731	68.190	68.182	69.371	1.062	0.132	67.650	69.565	72.278
Proportion of deceased (%)	1.409	16.857	41.018	19.131	3.664	3.050	0.608	0.738	2.171	4.735	6.619	100.000

2. The method of decomposing the differences between the life expectancies at birth elaborated and used in the Demographic Research Institute of the HCSO

When using this method first we calculate the number of the deceased in each age group of the life table based on the causes of death studied by using the elements of distribution of the deceased relying on the causes of death in reality or the composition by causes of death of the general age-specific death rates, which are sums of age- and cause-specific death rates

It is natural that $\sum_i {}_n d_{i,x} = {}_n d_x$; $\sum_x \sum_i {}_n d_{i,x} = l_0$; $\sum_x \sum_i {}_n d_{i,x} / \sum_{x=0} {}_n d_x = 1$ where i denotes the causes of death ($i = 1, ..., 11$).

The structure by the causes of death of the deceased in a life table is different from that of the deceased based on the causes of death in reality mainly because of the differences in the age structure of the real and the stationary life-table populations.

The differences between the life expectancies at age x can be calculated by using two methods. If we are interested only in calculating the differences between life expectancies at birth, the easiest way is perhaps first to calculate directly the mean age of victims of different causes of death, then those of all causes of death.

The mean age of death in different age groups may be calculated by using the formula

$$\bar{x}^{(M)} = x + \frac{{}_n L_x^{(M)} - n l_{x+n}^{(M)}}{{}_n d_x^{(M)}}$$

in the case of males and

$$\bar{x}^{(F)} = x + \frac{{}_n L_x^{(F)} - n l_{x+n}^{(F)}}{{}_n d_x^{(F)}}$$

in the case of females.

The mean age at death of all victims and those of different causes of death may be calculated by using the formulae in the case of males:

$$e_0^{0(M)} = \frac{\sum_{x=0}^{\omega} \bar{x}^{(M)} {}_n d_x^{(M)}}{\sum_{x=0}^{\omega} {}_n d_x^{(M)}} \quad \text{and} \quad e_{i,0}^{0(M)} \approx \frac{\sum_{x=0}^{\omega} \bar{x}^{(M)} {}_n d_{i,x}^{(M)}}{\sum_{x=0}^{\omega} {}_n d_{i,x}^{(M)}}$$

and in the case of females the formulae:

$$e_0^{0(F)} = \frac{\sum_{x=0}^{\omega} \bar{x}^{(F)} {}_n d_x^{(F)}}{\sum_{x=0}^{\omega} {}_n d_x^{(F)}} \quad \text{and} \quad e_{i,0}^{0(F)} \approx \frac{\sum_{x=0}^{\omega} \bar{x}^{(F)} {}_n d_{i,x}^{(F)}}{\sum_{x=0}^{\omega} {}_n d_{i,x}^{(F)}} .$$

If we divide the number of years the deceased lived by their mean ages, we obtain their numbers in the corresponding life table:

$$\frac{\sum_{x=0}^{\omega} \bar{x}^{(M)}\, {}_n d_{i,x}^{(M)}}{e_{i,0}^{0(M)}} \approx \sum_{x=0}^{\omega} {}_n d_{i,x}^{(M)} \quad \text{and} \quad \frac{\sum_{x=0}^{\omega} \bar{x}^{(M)}\, {}_n d_x^{(M)}}{e_0^{0(M)}} = \sum_{x=0}^{\omega} {}_n d_x^{(M)}$$

in the case of males and

$$\frac{\sum_{x=0}^{\omega} \bar{x}^{(F)}\, {}_n d_{i,x}^{(F)}}{e_{i,0}^{0(F)}} \approx \sum_{x=0}^{\omega} {}_n d_{i,x}^{(F)} \quad \text{and} \quad \frac{\sum_{x=0}^{\omega} \bar{x}^{(F)}\, {}_n d_x^{(F)}}{e_0^{0(F)}} = \sum_{x=0}^{\omega} {}_n d_x^{(F)}$$

in the case of females.

The most important question remains the same as what it was before: how to transform the differences between the age-specific mortality rates into differences between life expectancies at birth?

If we assume as before that $l_0 = 1$, it is clear that

$$l_x = exp\left[-M_x\right] = exp\left[-\int_0^x \mu_x d\,x\right] = exp\left[ln_x\, p_0\right] = exp\left[ln\, l_x\right]$$

where ${}_x p_0 = l_x/l_0 = exp[-M_x]$, i.e. the probability of surviving from birth till the exact age x and the number of survivors in the life table with $l_0 = 1$; μ_x denotes the value of the definite integral of the force of mortality within the limits of age groups, i.e. approximately the value of the age-specific mortality rate denoted generally m_x (or ${}_n m_x$) in life tables ${}_n m_x = (-ln_n\, p_x)/n$;

$$M_x = \int_0^x \mu_x dx = \sum_{x=0}^{x} {}_1 m_x = \sum_{x=0}^{x} n_x m_x = -ln_x\, p_0 = -ln\left(l_x / l_0\right)$$

If we consider the additivity of μ_x or m_x subdivided by causes of death, i.e. the fact that

$$\mu_{1,x} + \mu_{2,x} + ... = \sum_i \mu_{i,x} = \mu_x\,,$$

or

$${}_n m_{1,x} + {}_n m_{2,x} + ... = \sum_i {}_n m_{i,x} = {}_n m_x\,,$$

then we may write

$$e_0^{0(F)} - e_0^{0(M)} = \int_0^{\infty} \left\{exp\left[-M_x^{(F)}\right] - exp\left[-M_x^{(M)}\right]\right\} dx =$$

$$= \int_0^\infty \left\{ exp\left[-(M_{1,x}^{(F)} + M_{2,x}^{(F)} + ...)\right] - exp\left[-(M_{1,x}^{(M)} + M_{2,x}^{(M)} + ...)\right] \right\} dx =$$

$$= \int_0^\infty \left\{ \left[exp(-M_{1,x}^{(F)})\, exp(-M_{2,x}^{(F)}) ...\right] - \left[exp(-M_{1,x}^{(M)})\, exp(-M_{2,x}^{(M)} ...\right] \right\} dx =$$

$$= \int_0^\infty \left\{ exp\left[-\int_0^x \mu_x^{(F)} dx \right] - exp\left[-\int_0^x \mu_x^{(M)} dx \right] \right\} dx =$$

$$= \int_0^\infty \left\{ exp\left[-(\int_0^x \mu_{1,x}^{(F)} dx + \int_0^x \mu_{2,x}^{(F)} dx + ...) \right] - exp\left[-(\int_0^x \mu_{1,x}^{(M)} dx + \int_0^x \mu_{2,x}^{(M)} dx + ...) \right] \right\} dx =$$

$$= \int_0^\infty \left\{ \left[exp(-\int_0^x \mu_{1,x}^{(F)} dx)\, exp(-\int_0^x \mu_{2,x}^{(F)} dx) ... \right] - \left[exp(-\int_0^x \mu_{1,x}^{(M)} dx)\, exp(-\int_0^x \mu_{2,x}^{(M)} dx) ... \right] \right\} dx =$$

$$= \int_0^\infty \left\{ exp\left[ln_x\, p_0^{(F)} \right] - exp\left[ln_x\, p_0^{(M)} \right] \right\} dx =$$

$$= \int_0^\infty \left\{ exp\left[ln_x\, p_{1,0}^{(F)} \right] - exp\left[ln_x\, p_{1,0}^{(M)} \right] \right\} dx + \int_0^\infty \left\{ exp\left[ln_x\, p_{2,0}^{(F)} \right] - exp\left[ln_x\, p_{2,0}^{(M)} \right] \right\} dx + ... =$$

$$= \int_0^\infty \left[l_x^{(F)} - l_x^{(M)} \right] dx = \int_0^\infty \left[l_{1,x}^{(F)} - l_{1,x}^{(M)} \right] dx + \int_0^\infty \left[l_{2,x}^{(F)} - l_{2,x}^{(M)} \right] dx + ... =$$

$$= \sum_{x=0}^{\omega} \left[L_x^{(F)} - L_x^{(M)} \right] = \sum_{x=0}^{\omega} \left[L_{1,x}^{(F)} - L_{1,x}^{(M)} \right] + \sum_{x=0}^{\omega} \left[L_{2,x}^{(F)} - L_{2,x}^{(M)} \right] + ... = \sum_{x=0}^{\omega} \sum_i \left[L_{i,x}^{(F)} - L_{i,x}^{(M)} \right] =$$

$$= T_0^{(F)} - T_0^{(M)} = \left[T_{1,0}^{(F)} - T_{1,0}^{(M)} \right] + \left[T_{2,0}^{(F)} - T_{2,0}^{(M)} \right] + ... = \sum_i \left[T_{i,0}^{(F)} - T_{i,0}^{(M)} \right] =$$

$$= \sum_{x=0}^{\omega} \left[x\, d_x^{(F)} - x\, d_x^{(M)} \right] = \sum_{x=0}^{\omega} \left[x\, d_{1,x}^{(F)} - x\, d_{1,x}^{(M)} \right] + \sum_{x=0}^{\omega} \left[x\, d_{2,x}^{(F)} - x\, d_{2,x}^{(M)} \right] + ... = \sum_{x=0}^{\omega} \sum_i \left[x\, d_{i,x}^{(F)} - x\, d_{i,x}^{(M)} \right],$$

where 1, 2, ... etc. are the different causes of death, denoted previously by *i*.

John H. Pollard has shown that

$$e_0^{0(F)} - e_0^{0(M)} = \int_0^\infty \left\{ exp\left[-M_x^{(F)} \right] - exp\left[-M_x^{(M)} \right] \right\} dx =$$

$$= \int_0^\infty \left\{ exp\left[-M_x^{(M)} - M_x^{(F)} \right] - 1 \right\} {}_0 p_x^{(M)} dx =$$

$$= \int_0^\infty \left[\frac{{}_x p_0^{(F)}}{{}_x p_0^{(M)}} - 1 \right] {}_0 p_x^{(M)} dx =$$

$$\int_0^\infty \left[{}_x p_0^{(F)} - {}_x p_0^{(M)} \right] dx = \int_0^\infty \left[l_x^{(F)} - l_x^{(M)} \right] dx = \text{etc.}$$

His demonstration is undoubtedly correct, but the result we obtain by using his final formula is very different from our results.

If we are interested in calculating the differences between the life expectancies at higher ages too, we cumulate from the highest ages the values of ${}_n d_{i,x}$ for obtaining the numbers of survivors as future victims of different causes of death ($l_{i,x}$). It is obvious that $\sum_i l_{i,x} = l_x$ and the sum of the elements of the structures of survivors as future victims of different causes of death is equal to 1 at each exact age.

The next step is the calculation of the total stationary population and the stationary subpopulation in the life table by causes of death (${}_nL_{i,x}$). It is natural that

$$\sum_i {}_nL_{i,x} = {}_nL_x \ .$$

For the age intervals 0 to 1, 1 to 4 and 5 to 9, the calculation can be done by using the following well-known formulae:

$${}_1L_0 = (0{,}07 + 1{,}7M_0)d_0 + l_1, \text{ where } M_0 \text{ is the mortality rate for 0 year of age,}$$

$${}_4L_1 = 1{,}5\,{}_4d_1 + 4l_5, \quad \text{and} \quad {}_5L_5 = 2{,}5\,{}_5d_5 + 5l_{10}\,.$$

For the following five-year age intervals (until the age of 85) we have the formula:

$${}_nL_x = \frac{65}{24}(l_x + l_{x+5}) - \frac{5}{24}(l_{x-5} + l_{x+10})$$

For the last (open ended interval) the result may be obtained by using the following formula:

$${}_\infty L_{85} = l_{85} \cdot e_{85}^0 = l_{85}\,(1/{}_\infty M_{85}) = l_{85}/{}_\infty M_{85}\,,$$

where ${}_\infty M_{85}$ is the mortality rate for 85 years of age and above.

The calculation of the stationary subpopulation by causes of may be obtained supposing that

$${}_nL_{i,x} = n\,l_{i,x+n} + {}_nd_{i,x}\frac{{}_nL_x - nl_{x+n}}{{}_nd_x}\ ,$$

instead of

$${}_nL_{i,x} = n\,l_{i,x+n} + {}_nd_{i,x}\frac{{}_nL_{i,x} - nl_{i,x+n}}{{}_nd_{i,x}}\,.$$

Obviously this is not true; the distribution of the victims of different causes of death, especially in five-year age intervals, may differ from that of victims of all causes of death. More precise results may be obtained if the distribution of the deceased by causes of death for single-year intervals is available.

The next step is to calculate the total after-life time of all survivors and of the survivors as future victims with different causes of death. It may be realized by cumulating the ${}_nL_x$ and ${}_nL_{i,x}$ values from the highest ages and so $\sum_i T_{i,x} = T_0$.

The life expectancies at age *x* of all survivors and survivors as future victims with different causes of death may be calculated by using the formulae:

$$e_x^0 = \frac{T_x}{l_x} \quad \text{and} \quad e_{i,x}^0 = \frac{T_{i,x}}{l_{i,x}}\ .$$

Table 7

Contribution of mortality based on causes of death to the differences between life expectancies at birth of Hungarian females and males

The causes of death studied	Calculated using the method of				Calculated using the method of			
	Pollard	Andreev	Pressat	DRI of the HCSO	Pollard	Andreev	Pressat	DRI of the HCSO
	data for 1966				data for 1994			
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(9)	(9)
Infectious and parasitic diseases	0.341	0.342	0.341	-1.185	0.126	0.124	0.128	-0.014
Neoplasms(all forms)	0.640	0.677	0.647	-1.646	1.954	1.979	1.889	-1.339
Diseases of the circulatory system	1.068	1.056	1.056	5.881	2.503	2.518	2.457	8.616
Cerebrovascular diseases	0.169	0.180	0.175	3.532	0.658	0.665	0.641	2.678
Diseases of the respiratory system	0.324	0.339	0.333	-0.730	0.557	0.568	0.525	0.729
Diseases of the digestive system	0.138	0.134	0.136	0.240	0.199	0.193	0.204	0.229
Chronic liver disease and cirrhosis	0.101	0.099	0.101	-0.247	1.244	1.214	1.275	-1.479
Congenital anomalies	0.007	0.006	0.006	0.001	0.009	0.009	0.010	-0.051
Certain conditions originating in the perinatal period	0.450	0.427	0.440	-0.001	0.158	0.154	0.165	0.000
Injury and poisoning	1.282	1.227	1.267	-0.720	1.736	1.722	1.837	0.168
All other causes of death	0.234	0.267	0.252	-0.371	0.320	0.318	0.333	-0.073
Total	4.754	4.754	4.754	4.754	9.464	9.464	9.464	9.464

The life expectancy at birth ($e^0{}_0$), i.e. the mean age of all the deceased in the life table, as it has already been mentioned, is a weighted arithmetic mean of mean ages at the death of victims with different causes of death. If we denote the proportions of victims of different causes of death by f_i then

$$e_0^0 = \sum_i f_{i,0}\, e_{i,0}^0 \quad (\sum_i f_{i,0} = 1) \; .$$

The 'mean of means' nature of life expectancy at birth, or the mean age of all the deceased in the life table is sometimes presented by showing the balances with two hands. The weights hanging on both hands of balances are the numbers of the deceased in the life-tables due to different causes of death. Their sum is equal in our case to 100,000 (i.e. to the radix of the life-tables we use). The points of suspension of weights are the mean ages at the death of victims of corresponding causes of death. The sum of weights multiplied by the differences between the suspension points of weights and suspension point of balances is the same on both hands of the balances. The sign of these equal sums is nevertheless different and their algebraic sum is therefore equal to zero; corresponding to the concept of weighted arithmetic mean. The decomposition of the differences between the points of suspension of the balances means, in this case, the decomposition between the life expectancies at birth.

If we want to show not only the contribution of the different causes of death to the differences between the life expectancies at birth, but to present the contributions in question as the sums of 'structural effects' and 'mortality level effects' as well, we may use for this purpose the method of double standardization elaborated by *E.M. Kitagawa* (1955, 1964).

If we denote the weights when studying e.g. the differences between the life expectancies at birth of females and males by $f_{i,0}^{(F)}$ and $f_{i,0}^{(M)}$, and the life expectancies at birth of future victims with different causes by $e_{i,0}^{0(F)}$ and $e_{i,0}^{0(M)}$, then life expectancies at birth (exact 0 years of age) will be

$$e_0^{0(F)} = \sum_i e_{i,0}^{0(F)}\, f_{i,0}^{(F)} \quad \text{and} \quad e_0^{0(M)} = \sum_i e_{i,0}^{0(M)}\, f_{i,0}^{(M)} \; ,$$

and the difference between the expectancies at birth for females and males will be equal to

$$e_0^{0(F)} - e_0^{0(M)} = \sum_i [e_{i,0}^{0(F)}\, f_{i,0}^{(F)} - e_{i,0}^{0(M)}\, f_{i,0}^{(M)}] \, .$$

The contribution of mortality, based on different causes of death, to the differences between life expectancies at birth is very different from that calculated by using the methods of *Pollard* (1982, 1988), *Andreev* (1982), *Pressat* (1985, 1995) and *Arriaga* (1984). (See Table 7.) An explanation for the origin of these differences has been provided in two of the previous papers of *Valkovics* (1991, 1996).

In order to show the effect of the differences of the structures of the deceased based on causes of death and the effect of the differences of the mean ages in the death of victims with different causes of death in corresponding life-tables, we may use one of the following formulae:

$$e_0^{0(F)} - e_0^{0(M)} = \sum_i (f_{i,0}^{(F)} - f_{i,0}^{(M)}) e_{i,0}^{0(M)} + \sum_i (e_{i,0}^{0(F)} - e_{i,0}^{0(M)}) f_{i,0}^{(F)} =$$

$$= \sum_i (f_{i,0}^{(F)} - f_{i,0}^{(M)}) e_{i,0}^{0(F)} + \sum_i (e_{i,0}^{0(F)} - e_{i,0}^{0(M)}) f_{i,0}^{(M)} =$$

$$= \sum_i [f_{i,0}^{(F)} - f_{i,0}^{(M)}][0,5 (e_{i,0}^{0(F)} + e_{i,0}^{0(M)})] + \sum_i [e_{i,0}^{0(F)} - e_{i,0}^{0(M)}][0,5 (f_{i,0}^{(F)} + f_{i,0}^{(M)})] \ .$$

The first part of these formulae shows the impact of the differences of the structure of the deceased by causes in corresponding life-tables. The second part of these formulae shows the effect of the differences of mean ages of victims with different causes of death in corresponding life-tables.

We emphasise that double standardization method may only be used for decomposing the contributions of different causes of death to the differences between life expectancies at birth into 'structural effects' and ‘mortality level effects’.

When comparing two mortality structures by causes of death (in other words: two structures of the deceased by causes of death) we can see that the mortality structure which is more favourable from the point of view of the mortality level is the one where the proportion of causes of death killing their victims at older ages is higher. When we compare two sets of mean ages of victims of different causes of death, the set with higher mean ages is more favourable. A more favourable mortality structure and a more favourable set of mean ages at the death of victims with different causes of death result a higher life expectancy at birth, i.e. a lower mortality level and vice versa.

The observed mean ages at death and the mean ages at death in the life tables by causes of death of victims of different causes of death we use in our contribution are naturally not independent, they are influenced by the fact that each cause of death is acting in coexistence with all the other causes of death. I few special cases, when it is possible to calculate them in pure state, as every demographer knows it, the non-independent mean ages may be even very different from the independent mean ages.

If we consider the method elaborated and used in the Demographic Research Institute of the HCSO we must focus on the influence of rising or diminishing proportions of victims of a given cause of death in the life table death function on the diminishing or rising proportions of victims of other causes of death which contribute also the rise or decline of general mortality level as well.

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