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Experimental designs for radiation dosimetry calibration

Topic 2 – Learning more from what we already know

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Introduction

The tools provided by optimal design of experiments theory are applied to a nonlinear calibration model. This is motivated by the need of estimating radiation doses using radiochromic films. The calibration model is in this case nonlinear and the explanatory variable cannot be worked out explicitly from the model in general.

The main problem is that here a design has to be found on the dependent variable. For that, the inverse function theorem will be used to obtain an information matrix to be optimized. We compare in a particular case the proper D–optimal design on the response variable with the transformed design in the explanatory variable.

Moreover, c–optimal designs are also computed and used to calculate the efficiency of the D–optimal design to estimate each parameter. In order to provide designs with more support points, suboptimal designs are also computed when the support points are forced to follow some mathematical rule. Finally GI– and VI–optimal designs are computed to optimize the inverse prediction of the calibration model, which actually is the main objective of the calibration problem.

Conclusions

By using the Inverse Function Theorem optimal and suboptimal designs were computed on the dependent variable for estimating the parameters of the model and for the prediction of the independent variable. The D–optimal design was computed directly on the explanatory variable and then it was transformed into a design on the response variable.

This design displayed a moderate loss of efficiency in our example when compared with the right one. Optimal designs for estimating each parameter of the model were also computed. This allowed to measure how efficient was the D–optimal design for estimating each of then giving good efficiencies in all the cases. Since thee–point designs may be not acceptable from a practical point of view ten different points were forced to be in the design forcing them to follow a regular sequence. In particular, arithmetic, geometric and their inverses (through the trend model) sequences were considered. All of them were more efficient than the sequence used by the researchers. The geometric sequence got a high efficiency.

Finally, taking into account that this model has calibration proposes, the GI– and VI–optimal designs has been computed in order to optimize the inverse prediction. Moreover, the algorithms for computing them are detailed.