EKS INDEX AND INTERNATIONAL COMPARISONS*

PÁL KÖVES[1]

SUMMARY

The *EKS* (*Éltető – Köves – Szulc*) index formula (hereinafter referred to as *X*) is in official use in the comparison of per capita real GDP and purchasing power parity of the OECD countries. Early versions of formula *X* can be found in Fisher's and Gini's works.

According to the author, *X* can be regarded as the result of a multi-situational crossing. It is the geometric mean of the 'generalized *F*' and its factor-antithesis. The three-member index family obtained this way shows a close relationship to *J. van Yzeren's* three indices (family **Y**). The paper presents an empirical procedure to measure the similarity of the behaviour and performance of the different index formulae. The author comes to the conclusion that the members of families **X** and **Y** walk around index *X*.

KEYWORDS: Index formulae; International comparisons.

Different index formulae are used to measure the real value of GDP and the temporal change of price level. If the target is not the measurement of temporal change but the spatial comparison of different countries for a given period (year) then a special use should be considered when choosing the index formula. We know many volume-index formulae that are suitable for comparing the real value of per capita GDP of a fixed number (m) of countries. Consequently there are many price-index formulae that can be used to compare the purchasing power of currencies of different countries.

One of the index formulae used for international comparison is *EKS* volume and price index which was introduced separately but at the same time by the two Hungarians *Éltető* and *Köves* (1964) and the Polish *Szulc* (1964). Both propositions use Fisher's (F) bilateral 'ideal' non-transitive indices to produce transitive indices.[2]

$$X_{t/b} = \left(\prod_{i=1}^{m} F_{t/i} \cdot F_{i/b}\right)^{\frac{1}{m}} = \left(F_{t/b}^{2} \cdot \prod_{i \neq t/b}^{m} F_{t/i} \cdot F_{i/b}\right)^{\frac{1}{m}} \qquad /1/$$

* The abridged and revised version of article *Köves* (1995) originally published in Hungarian.

[1] Professor Emeritus.

[2] Transitivity means that equations of type $I_{2/1} \cdot I_{3/2} = I_{3/1}$ hold.

In the index

$$F_{t/b} = \left(L_{t/b} \cdot P_{t/b}\right)^{\frac{1}{2}} \qquad /2/$$

is the geometric mean of the respective bilateral Laspeyres (*L*) and Paasche (*P*) indices.[3]

It was proved by *Éltető* and *Köves* (1964) that $X_{t/b}$ also meets the requirement

$$\sum_{t=1}^{m}\sum_{b=1}^{m}(\log F_{t/b} - \log X_{t/b})^2 \to \min. \qquad /3/$$

This means that replacing the non-transitive indices $F_{t/b}$ by the transitive $X_{t/b}$ indices minimizes the overall replacement error in a certain sense. The relative magnitude of the indices are represented by their logarithms. /3/ requires, that the sum of squares of differences between the logarithms of *F* and *X* indices should be minimum for all indices calculated from all *t*/*b* relation. Formula /1/ means that country *t* and country *b* are compared sequentially through the mediation of each county (i=1,…m) and then the geometrical mean of the indirect results is considered as a final result. Countries *t* and *b* are also taken as 'mediator' countries, consequently the direct comparison carries a double weight in the comparison.

To justify formula /1/ Szulc states that a mean value of the direct and indirect comparisons should be calculated. His initial formula differs from the second form of /1/ in considering the direct comparison $F_{t/b}$ on the first power. To assure the transitivity of these results an iterative infinitesimal process is suggested which approaches the indices $X_{t/b}$.

In addition to transitivity, *X* also passes one of the most important index tests, the time reversal and factor reversal tests.[4] It does not pass the proportionality test,[5] but the degree of its violation is negligible in practice. (This was proved by *Köves* (1995) on pages 23-26, by giving both some theoretical and numerical arguments.)

The absence of additivity is usually considered a disadvantage. (The sum of values corresponding to the partial indices does not equal to the value corresponding to the general index.)

In the first phase of ICP an experimental comparison has been made where the 1970 figures of ten countries and four index calculation methods have been used (*Kravis* et al., 1975). One of these was formula *X*, as it appears in /1/. It was named *EKS* after the authors *Éltető* and *Köves* (1964) and *Szulc* (1964).

[3] Here $L_{t/b}(t,b=1,\ldots m)$ which relates country *t* to country *b*, can stand for the volume index $L^q_{t/b} = \sum p_b q_t / p_b q_b$ or the price index $L^p_{t/b} = \sum p_t q_b / \sum p_b q_b$ (purchasing power parity). (Here and in all further formulae the summing up limits by commodities will be neglected.) Similarly, $P_{t/b}$ can be made concrete by volume index $P^q_{t/b}$ or price index $P^p_{t/b}$. (For numerical examples see Appendix II.)

[4] The time reversal test requires the equation $I_{t/b} \cdot I_{b/t} = 1$ to be valid, that is the indices calculated with a given formula by reversing the two periods of time must be in reciprocal relationship. The factor reversal test requires that the product of the volume and price indices be equal to the value index $V_{t/b} = \sum p_t q_t / \sum q_b p_b$.

[5] The proportionality test (commonly referred to as average test) requires that the aggregate index be $I=c$ if each individual index equals to a constant c.

The three other methods used were

1. the Geary–Khamis method *GK* [*Khamis* (1972), *Kravis* et al. (1975)],
2. the Walsh price index *W* [*Kravis* et al. (1975)],
3. J. van Yzeren's 'balanced' method *Y* [*Yzeren* (1956)].

All four methods are results of some multi-situational crossing (*Köves*, 1983 a, b). Only *GK* provides additive results because its calculation assumes the existence of certain average prices. Price index *W* is arrived at by calculating the weighted geometrical mean of the individual indices, where the weight attached is the arithmetical mean of the value shares of the smallest units over the countries. The *W*-type volume index is the ratio of the value index and price index *W.* Index *W* could only be made transitive by fixing the basis (USA).

The common data set of *X* and *Y* is a matrix which contains the volume indices in all relations. If m=2, both methods produce the Fisher index. The numerical results of *X* and *Y* are very close to each other, but *X* has a direct formula, while the *Y* results can be obtained by an iterative process, similarly to *GK*.

Drechsler,[6] the well-known Hungarian ICP expert played a major role in recommending to name formula *X EKS*. As one of the three persons whose initials produce *EKS*, I must mention that the birth of the *Éltető* and *Köves* (1964) paper should also be attributed to Drechsler's inspiration or 'order'. An important step in this process was the publication of Drechsler's book (1962), the appendix of which written by Éltető, contains formula /1/.

The afore-mentioned 'competition' of the four formulae was won by *GK* and thus it was used to produce all published indices of the first five ICP phases. Its recognition was due, on the one hand, to the fact that its equation system reflected attractive economic considerations and that on the other hand, it met the requirement of additive consistency. Not much later, however, it was considered a growing disadvantage that *GK* volume index systematically appreciated countries with a low GDP, and the price index did the same with the currencies of these countries.

After the regionalisation of ICP in 1980, a separate European comparison has been carried out. ECP'90, began to use primarily *EKS*[7] in the view of the criticism raised against GK. In addition to the official *EKS* results, *GK* results appeared only as secondary figures[8]. (See *Hill*, 1997; *Khamis*, 1984; *Khamis*, 1996; *Prasada*, 1997 too.)

1. Irving Fisher and formula *X*

Fisher (1922) considered the time reversal test and the factor reversal test of utmost importance. The reason why these two tests are so important for Fisher is that they have a 'formula discovering' function. The tool of discovery is the antithesis index, and the result is a new crossed index, which meets a requirement that the original index does not.

If we calculate an index by some formula *I* not passing the time reversal test, by reversing the two time periods, the reciprocal of this is the time-antithesis. The geometrical

[6] *L. Drechsler* was the director of ICP in 1985–1989. He died in 1990.
[7] Purchasing Power Parities and Real Expenditures. *EKS* Results, 1990. OECD. Paris. 1992.
[8] Purchasing Power Parities and Real Expenditures. *GK* Results, 1990. OECD Paris. 1993.

mean of the original index and the antithesis is then the crossed index. If we calculate a volume index by using a price index formula not passing the factor reversal test, and then the value index is divided by this index, we obtain the factor antithesis. The geometrical mean of the original price index and the antithesis is the crossed index. (This is shown in Table App. 1.)

Fisher (1922) states on page 416. that any index formula that passes the two most important tests can be modified so as to pass the circularity test as well, so that the formula be transitive for three periods of time or spatial units. The method can also be extended to more than three situations.

The modification to be made is

$$I'''_{2/1} = \frac{I''_{2/1}}{(I''_{2/1} \cdot I''_{3/2} \cdot I''_{3/1})^{\frac{1}{3}}} \qquad /4/$$

where

I is the initial formula,
I' is the index crossed with factor antithesis,
I'' is the index crossed with both antitheses, and
I''' is the index which also passes the circularity test, in addition to the other two ones.

The point here is that in two special cases (if I=L or I=P) I''' will be identical to X for three countries. What Fisher stresses is that circularity (transitivity) can be attained by applying a multi-stage process to any formula. Thus, we can say that Fisher produced the earliest version of the *EKS* formula.

2. Corrado Gini and formula *X*

Gini (1924 and 1931) proposed several transitive index formulae for the purpose of spatial comparisons. His propositions included both the generalised Edgeworth–Marshall-type price index

$$\overline{E}^{p}_{t/b} = \frac{\sum\sum_{i=1}^{m} p_t q_i}{\sum\sum_{i=1}^{m} p_b q_i}, \qquad /5/$$

and the generalised Fisher index

$$\overline{F}^{p}_{t/b} = \left(\prod_{i=1}^{m} \frac{\sum p_t q_i}{\sum p_b q_i}\right)^{\frac{1}{m}}. \qquad /6/$$

(For numerical illustrations see Appendix II.) Another proposed formula is:

$$\overline{I} = \left(\prod_{i=1}^{m} \frac{I_{t/i}}{I_{b/i}}\right)^{\frac{1}{m}} \qquad /7/$$

If I is replaced by F in /7/, we will have X. Since Gini (just like Fisher) did not state requirement /3/, he could not possibly attach a 'high rank' to /1/. At the same time, it seemed natural for him that the better the underlying two-situational I was, the better /7/ became. This is also shown by the fact that in his numerical example he made I concrete by using F.

Gini (1931) compared the prices of five Italian cities in eight periods of time. He calculated the price index for all relations by applying 14 different formulae. This was much to the delight of researchers (also) interested in the empirical testing of index formulae. One of these was X, so the first application of *EKS* can be found in *Ginis'* paper. Several decades had to pass until the next application.

Italian authors *Biggeri et al.* (1987) propose that the name of X should begin with a letter G. It is of course a reasonable wish, however one must not forget about Fisher either. (An overview of the history of formula X is in Appendix I.)

3. An obvious derivation of formula *X*

Requirement /3/ or the iterative procedure suggested by *Szulc* (1964) is not necessarily the most natural way of developing formula X. Gini's formula signals the route from two-situational indices to X. Gini gave formula /7/, which is more general than X, since he left out a link in the chain of deriving it. This missing link is formula /8/. On page 249. *Szulc* (1964) gives the entire logical route between F and X. (See also *Köves*, 1975, p. 1199. and *Köves*, 1983, p. 150.) The multi-situational generalisation of F produces $\overline{F}$. Since $\overline{F}$ does not pass the time reveral test, I produce its factor antithesis by dividing the value index by the volume index corresponding to price index formula /6/:

$$\overline{F}_{t/b}^{pa} = \frac{V_{t/b}}{\overline{F}_{t/b}^{q}} = \frac{V_{t/b}}{\left(\prod_{i=1}^{m} \frac{\sum p_i q_t}{\sum p_i q_b}\right)^{\frac{1}{m}}} \qquad /8/$$

The geometric mean of the original formula /6/ and its antithesis /8/ is then

$$X_{t/b}^{p} = \left(\overline{F}_{t/b}^{p} \cdot \overline{F}_{t/b}^{pa}\right)^{\frac{1}{2}} \qquad /9/$$

which is identical to /1/. (See the numerical example in Appendix II.)

It must be mentioned here that out of these three indices price index /6/ and the cor-res-ponding volume indices pass the proportionality test because they are the means of aggregate indices, with different weights but identical price relatives, which pass the test. In antithesis /8/ the guarantee of passing the test vanishes, and formula /9/=/1/ inherits this deficiency.

4. Two families of index formulae

J. van Yzeren (1956) described three closely related methods of which we have only touched method III or the balanced method. These three methods are closely related to the triad /6/, /8/ and /9/. This relationship can also be expressed by the symbols used. The indices of family **Y** are Y', Y'' and Y, while family **X** consists of X', X'' and X. Table App. 4 contains the equations referring to country j (j=1, …m) of the corresponding equation systems.

The equations combine the non-transitive Laspeyres indices (L) with transitive exchange rates. The first two methods are slightly biased, which is balanced by the third one. If any of the three methods is applied to two countries, $Y' = Y'' = Y = F$, and the same holds for family **X**.

Here we can again demonstrate the close relationship between the two families. If the expressions with the summation sign in Table App. 4 are replaced by their logarithms, the operations will give the logarithms of the indices of family **X**. Consequently, a direct formula can be given instead of the iterative process.

Let us take out the indices of the two families corresponding to relation *B*/*A* from Table App. 3:

Y''		Y		Y'	X''		X		X'
1.0076	<	1.0098	<	1.0120	1.0075	<	1.0097	<	1.0118

It is not accidental that the results of method III are in the centre, and that the order of magnitude for the relation picked is identical in both families.

W. D. Heller (1982) points out that the matrix of empirical indices which is necessary to calculate the 'van Yzeren-type' indices may consist not only of L, but also of P or F indices.

In my paper (*Köves*, 1995. p. 20–22), I completed Heller's calculations and consideration, and extend the investigation to family **X**. The conclusions can be summarised as follows.

If we use matrix P instead of matrix L for calculating the indices of family **Y**, replacing Y' by Y'' it would produce similar results as matrix L. If this exchange of matrices is made for family **X**, we obtain not only similar but the same results by replacing method I by method II. If this is done with matrix F, then $X' = X'' = X$.

The function of the contrasting pairs is that one member relies on the proportionality test in terms of prices, while the other in terms of quantities. The 'balanced index' (in both families) produces a satisfactory situation in both respects.

A further conclusion is that the duality of volume and price indices is added to the duality of methods I and II, and matrixes L and P. (Thus far, we have always dealt with the price index only – in a direct way.) We can also speak of the duality of positive or negative correlation between the volume and price relatives.

An even shorter summary is that X stands in the focus of the narrower and broader system of 'van Yzeren-type' indices and the elements of the system 'are dancing around' it.

5. Shall we weight the weights? Further formulae

In the first phase of ICP *GK* was chosen because the additive relations can be retained by taking 'international average prices' to calculate the volume index. The term 'international average price' seems reasonable since it is calculated as the weighted mean of the converted national prices. It appears that attaching different weights to the weights also deserves recognition: the average price appears more realistic. However, the weights of the volume indices are not the prices but the relative prices. (The relative prices of large and small countries should be considered equal if we are to compare them by index numbers.)

In index calculation the absolute price merely ‘wears’ the relative price as a model wears a dress. If the index expert selects a ‘good’ price and not a ‘good’ relative price, he acts as a designer who selects the most beautiful model instead of the most beautiful dress. Weighting in the process of averaging price weights is not the implementation of some economic requirements but the order of the model which includes arbitrary elements as well.

The Geary-Khamis index has two shortcomings. The first one is a common property of average-price indices: the bias due to the negative correlation between the volume and price relatives (the so-called Gerschenkron effect). It is well-known, that $L>P$ in case of negative correlation. The price structure of some countries is close to the average, while that of others is very far from it. Thus, the index of the former countries will be quasi P, while that of the latter will be quasi L. The other shortcoming of GK can be explained with weighting the weights.[9]

Of the average-price methods Gerardi’s formula (*Ge*) (*Gerardi*, 1982; *Köves*, 1983 a p. 156.) embodies the principle of unweighted averaging the most consistently. The weights of the *Ge* volume index come into being as a simple geometric mean of unconverted unit prices expressed in different currencies. The price index is the quotient of the value index and the volume index.

If any two-situational crossed formula (e.g. F) does not require average prices, there may be a hidden but verifiable average price in the background. (This is shown for F by van *Yzeren*,1952.) For multi-situational formulae that do not pass the average test, the average prices cannot be calculated, and they do not even exist. Only the price level is fixed for the aggregates given in publications.

Of the average-price formulae, attention must be made of Iklé’s formule (*Ik*), which can be calculated from a model similar to *GK* applying an iterative method. Here, however, weighting the weights is much more fortunate. (See *Köves*, 1983 b).

In *Balk’s* (1996) calculations, the weighted versions of X and Y also appear. (In this case value weights referring to countries rather than weights within the weights are used.) The unjustifiable weighting only slightly deteriorated the quality of the ‘unweighted’ X and Y (see *Köves*, 1995. p. 16.). I think I may discard these results.

6. The competition of index formulae

Table 5 in Appendix II. shows the purchasing power parities taken from the results of the first ICP phase (*Kravis et al.*, 1975).

So that we can assess the similarity between the results obtained by different formulae, we calculated a correlation coefficient from the logarithms of the parities produced

[9] Suppose, we calculated volume indices from the data of many countries, which were weighted by average prices but were free from the special bias of formula *GK*. If these values are plotted against the values obtained by some good index formula (e.g. *EKS*) then the points will approach a convex parabola of second degree. Countries could be found with an ‘average’price structure around the minimum point of the parabola. The special bias of the *GK* formula would be expressed by a declining line. Thus the joint effect of the bias of these two kinds could be described by a combination of the parabola and the line.

by each pair of the formulae and then subtracted their squared values from 1 (residual component). Table 1 shows these results[10] multiplied by 10^6.

Table 1

Residual components referring to all pairs of the formulae, $10^6 \cdot (1-r^2)$

Formulae	Symbol	*GK*	*W*	*X*	*Y*	Total
Geary-Khamis	*GK*	–	721	565	571	1857
Walsh	*W*	721	–	153	161	1035
EKS	*X*	565	153	–	1	719
van Yzeren	*Y*	571	161	1	–	733

The strikingly closest relationship is between *X* and *Y*, while *GK* and *W* are the furthest apart. The last column shows the totals of the figures in each row (or column), which reflects how similar numerical result each formulae produced in relation to the others. The first two places are taken by *X* and *Y*.

J. van Yzeren (1987) illustrated the indices that he had proposed with a further schematic example, which is similar to the one contained in *Yzeren* (1956). *Balk* (1996) calculated a series of further indices using the figures of this example. Appendix II includes some of the calculations made by the two authors. (*Köves*, 1995 presents all the results together with their residual components.)

Table 2 shows the most important 'competition results' of the 9 formulae, which I regard as the most important ones.

Table 2

Residual components, $10^6 \cdot (1-r^2)$

Formula	*Y*	*X*	*Y′*	*Y″*	*X′*	*X″*	*Ge*	*Ik*
GK	665	662	854	505	848	504	589	801
Y		0	13	13	12	13	8	7
X			13	12	12	12	7	8
Y′				50	0 a	51 a	40 b	6 b
Y″					49 a	0 a	1 b	31 b
X′						49	39 c	5 c
X″							1 c	34 c
Ge								26

It can be seen that the relationship of *GK* to the others is strikingly bad. We can also see that *X* and *Y* are the closest to each other: under the given accuracy the corresponding residual could be rounded to zero, i.e. the correlation coefficient to 1. The two rounded residual figures which characterize the relationship between members of families **X** and **Y** in the same position are also zero. The two zeros are shown on the diagonal line in box

[10] In the calculation of the coefficient between *GK* and *W* the first pair of values is: lg8.01, lg8.76. The value of the coefficient is: 0.9996395. This produces 721 as residual variance if multiplied by 10^6.

'a'. The other diagonal is taken by relatively high values. This means, the one-prime member of one family 'does not like' the two-prime member of the other.

Boxes 'b' and 'c' are almost identical. The one-prime members of both families show a close relationship with formula *Ge*, while the two-prime members have a slightly looser relationship with formula *Ik*. The 'other' diagonal also reflects a fairly friendly relationship.

I think *Ge* and *Ik* must be taken into account in addition to *X* when the next application is considered. However, in the light of the test calculations formulae X' and X'' can also be regarded as leaders. Table 3 may give some help to the consideration. (The residuals are taken from Table 2.)

Table 3

Comparison of five price index formulae

Formula	Symbol	Residual sum	Way of calculation	Proportionality test		Additivity
				price	volume	
EKS	X	393	direct	no	no	no
Gerardi	*Ge*	800	direct	no	yes	yes
Iklé	*Ik*	805	iteration	no	yes	yes
Generalised *F*	X'	1171	direct	yes	no	no
Antithesis of X'	X''	1085	direct	no	yes	no

The term 'direct' can, of course, mean a simple or a complex calculation. The numerical extent of not passing the test and of the lack of additivity can also differ.

Just like in the first phase of ICP, it seems advisable to implement an experimental phase again.

APPENDIX I

AN OVERVIEW OF THE HISTORY OF FORMULA *X*

1. Fisher (1922) develops an adjustment formula that guarantees circularity for three situations, which leads to *X* when starting from formula *L* or *P*.

2. C. Gini (1924) creates the generalised F ($\overline{F}$).

3. C. Gini (1924) constructs a general crossed formula, which gives *X* if the initial formula is *F*.

4. C. Gini (1931) publishes the results of his calculations obtained with 14 formulae, including *X*.

5. Ö. Éltető (in the Appendix of *L. Drechsler's* book, published in 1962), unaware of the antecedents in 1–4., introduces formula *X* by an intuitive explanation.

6. B. Szulc (1964) creates the antithesis of *F* and *X* by crossing. (Items 6-8 were published in independent studies, which appeared at the same time.)

7. B. Szulc (1964) shows an iterative method leading to *X* using an intuitive reasoning (and infinitesimal verification).

8. Ö. Éltető and *P. Köves* (1964) establish the minimum square property of *X*.

9. ICP makes the 1970 comparisons experimenting with four formulae. One of the four formulae is *X*, which here gets the name *EKS*.

10. P. Köves (1975) gives a general overview of multi-situational crossing, and places *X* in this context. He reveals the close relationship between 'index families' *X* and *Y*. (The symbol *X* originates from this paper.)

11. OECD's official results for 1990 are produced by using the *EKS* formula.

APPENDIX II
TABLES, CALCULATIONS

1. Crossing the index formulae

Table App. 1

Two types of crossing

Test	Original index	Antithesis	Crossed index	Check of the test
Time reversal	$I^p_{t/b}$	$\frac{1}{I^p_{b/t}}$	$\left(I^p_{t/b}\cdot\frac{1}{I^p_{b/t}}\right)^{\frac{1}{2}}$	$\left(I^p_{t/b}\cdot\frac{1}{I^p_{b/t}}\right)^{\frac{1}{2}}=\frac{1}{\left(I^p_{b/t}\cdot\frac{1}{I^p_{t/b}}\right)^{\frac{1}{2}}}$
Factor reversal	$I^p_{b/t}$	$\frac{V}{I^q_{t/b}}$	$\left(I^p_{t/b}\cdot\frac{V}{I^q_{t/b}}\right)^{\frac{1}{2}}$	$\left(I^p_{t/b}\cdot\frac{V}{I^q_{t/b}}\right)^{\frac{1}{2}}\left(I^q_{t/b}\cdot\frac{V}{I^p_{t/b}}\right)^{\frac{1}{2}}=V$

2. A numerical example

The following example has been elaborated by van Yzeren and shows fictional data of 4 countries (*A, B, C, D*).

Table App. 2

$\sum p_i q_j$ *data for four countries*

i \ j	*A*	*B*	*C*	*D*
A	5 800	27 175	1 206	1 396
B	5 950	26 925	1 234	1 407
C	74 240	345 200	12 108	14 144
D	15 570	71 175	2 490	2 718

Source: *Yzeren* (1987).

Using the data of Table App 2 formulae /1/–/9/ are computed as follows:

/1/ $$X_{B/A}=\left[1.0082^2\cdot(0.08916\cdot 11.3362)\cdot(0.4425\cdot 2.2862)\right]^{\frac{1}{4}}=1.0097\ ,$$

where

/2/ $L^p_{B/A}=5950/5800=1.0259$ $P^p_{B/A}=26925/27175=0.9908$ $F^p_{B/A}=(1.0259\cdot 0.9908)^{\frac{1}{2}}=1.0082$.

/5/ $$\overline{E}^p_{B/A}=\frac{5950+26925+1234+1407}{5800+27175+1206+1396}=0.99829\ .$$

/6/ $$\overline{F}^p_{B/A}=\left(\frac{5950\cdot 26925\cdot 1234\cdot 1407}{5800\cdot 27175\cdot 1206\cdot 1396}\right)^{\frac{1}{4}}=1.01184\ .$$

/8/ $$\overline{F}^{q}_{B/A} = \left(\frac{27175 \cdot 26925 \cdot 345200 \cdot 71175}{5800 \cdot 5950 \cdot 74240 \cdot 15570} \right)^{\frac{1}{4}} = 4.60748 \qquad V_{B/A} = \frac{26925}{5800} = 4.64224,$$

therefore

$$\overline{F}^{pa}_{B/A} = \frac{4.64224}{4.60748} = 1.00754 \ .$$

/9/ $$X^{p}_{B/A} = (1.01184 \cdot 1.00754)^{\frac{1}{2}} = 1.0097 \ .$$

Table App. 3

The purchasing power parities (price indices) of currencies of four countries obtained by different index formulae from the data of Table App. 2

Formula	*B/A*	*C/A*	*D/A*
F	1.0082	11.3362	2.2862
GK	1.0166	10.3526	2.0387
Y	1.0098	11.3689	2.2767
X	1.0097	11.3698	2.2760
Y'	1.0120	11.3293	2.3062
Y''	1.0076	11.4098	2.2478
$X' = \overline{F}$	1.0118	11.3405	2.3057
$X'' = \overline{F}^{a}$	1.0075	11.3995	2.2467
GE	1.0078	11.4142	2.2543
Ik	1.0058	11.6295	2.3036

Source: *Yzeren* (1987), *Köves* (1995), *Balk* (1996).

The indices F^{p} not given in Table App. 3:

$$F^{p}_{B/C} = 0.08916 \qquad F^{p}_{B/D} = 0.4425 \qquad F^{p}_{C/D} = 5.0303$$

3. Two families of index formulae

Table App. 4

The comparison of families **X** *and* **Y**

Method	Family **Y**			Family **X** Direct formula
	Equations	Name in Yzeren (1956)	Name in Yzeren (1987)	
I.	$\sum_{i=1}^{m} L_{j/i} \frac{Y'_i}{Y'_j} = s$	Method of heterogeneous groups	*q*-combining method	$X' = \overline{F}$
II.	$\sum_{i=1}^{m} L_{i/j} \frac{Y''_j}{Y''_i} = s$	Method of homogeneous groups	*p*-combining method	$X'' = \overline{F}^{a}$
III.	$\sum_{i=1}^{m} L_{j/i} \frac{Y_i}{Y_j} = \sum_{i=1}^{m} L_{i/j} \frac{Y_j}{Y_i}$	Balanced method	Balanced method	$X = \left(X' \cdot X'' \right)^{\frac{1}{2}}$

4. Empirical comparison of index formulae

Table App. 5

Purchasing power parity indices in 1970 applying four formulae (United States = 1)

Country	Currency	Geary–Khamis *GK*	Walsh *W*	*EKS* *X*	van Yzeren *Y*
Columbia	P	8,01	8,76	8,42	8,41
France	Fr	4,48	4,46	4,35	4,33
Federal Republic of Germany	DM	3,14	3,24	3,16	3,16
Hungary	Ft	16,07	15,92	15,93	15,90
India	Re	2,16	2,46	2,47	2,47
Italy	L	483	470	457	457
Japan	Y	244	247	240	239
Kenya	Sh	3,74	4,17	3,80	3,79
United Kingdom	£	0,308	0,291	0,291	0,291

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