EXPS FOR WINDOWS, A SOFTWARE APPLICATION *

TIBOR KISS[1] – BÉLA SIPOS[2]

SUMMARY

Exponential smoothing is still very popular world-wide. Well-known and frequently used statistical program packages contain this methodology. This paper demonstrates an exponential smoothing program that provides a more sophisticated software solution for this statistical method, namely it allows the automatic selection of the smoothing type, and it calculates a 'what if' and a sensitivity analysis.

Keywords: Exponential smoothing; Program packages.

Some kind of forecasting has always been a need of companies who wanted to know the near future. From the wide range of statistical methods decision makers have to find the one that fits best their actual situation. In situations where decision makers wanted to predict the continuation of a problem or relationship or wanted to forecast changes, time series methods were applied. Since the early 60s, with the growth in size and complexity of companies, the need for more and more sophisticated time series methods has increased. Computer usage spread from the early 70s, and time-shared computers were available at organisations. This spread of computers still continues. *Makridakis* et *al.* (1983, p.14) stated that in the 80s the greatest gains would derive from application and not new methods. New methods that are currently in the main stream are really important. These methods include: chaos theory (anharmonic analysis), *Gleick* (1987), wavelet analysis, *Percival* and *Harold* (1997) and others. However, a number of researchers are still working on exponential smoothing (*Aerts* et *al*., 1997; *Cleveland* and *Loader*, 1996; *Efron* and *Tibshirani*, 1996; *Eilers* and *Marx*, 1996; *Fan* et *al*., 1996; *Hardle* and *Marron*, 1995; *Jones*, 1996; *Jones* and *Foster*, 1996; *Marron*, 1996; *Wahba* et *al*., 1995). It is known world-wide and is effective; especially for short term forecasting purposes. Compared with other methods, like the Box-Jenkins method, exponential smoothing often has superiority (*Makridakis* et *al*., 1983). However, the

* This study was written in the framework of 030960 and 023058 program of Hungarian Scientific Research Fund (OTKA). This is a part of the research project of Széchenyi and MÖB scholarship of Professor *Béla Sipos*.

[1] Associate professor of the Janus Pannonius University of Sciences.
[2] Doctor of economic sciences, general vice-rector of the Janus Pannonius University of Sciences.

number of computers, currently available and the time shortage of decision makers emphasize the importance of software applications that use the ‘old’ methodology.

Exponential smoothing is a relatively simple method. It does not need a profound mathematical-statistical background, but it can provide a useful information base for decision making. It has the best performance in the case of short-term forecasting, e.g. monthly or weekly data (*Makridakis* et *al*., 1983). Most current statistical packages (SPSS, Statistica) also comprise the exponential smoothing method. Software developers are compelled to design better and better quality products. Software that can satisfy consumers’ needs is very important. The time interval between two versions of leading software packages has decreased to less than 1 year.

1. Theory

This section provides the theoretical background of the proposed computer program. First the most important features of the time series will be described followed by a very brief summary of the methodology of exponential smoothing.

Time series

If decision makers want to know more about the future, they have to collect data from the past. This time series is used for forecasting patterns derived from data. It is necessary to know the past behaviour of the process (if it is possible) in order to recognise its future. A basic principle of forecasting is to project the connection with the help of the knowledge of past and present data.

The simplest tools of time series analysis are computation of ratios and delineation of time series. Delineation is a useful tool, because it makes possible to recognise the type of function and constant trend. With mathematical-statistical methods one can do a more profound analysis, because the knowledge of deeper processes and principles can help with extrapolation.

Time series are always the results of observations, and researchers have to recognise principles on the basis of these data. Nowadays the fast change in economy results in time series a lot of breakpoints; sometimes a continuous length of time series is 5-6 years or less. However, it is not always possible to prepare correct extrapolation because of sudden changes. A more complex economic analysis is needed to determine the probability of unchanged variables or a variable for which the variation can be calculated.

It is necessary to have an appropriate length of time series - it is said to be as long as the extrapolated length (*Makridakis* et *al*., 1983). For a truly sound extrapolation about a minimum of six-year series is necessary (*Makridakis* et *al*., 1983), where e.g. the first three years can be computational periods (testing and estimating the seasonal component, because seasonal fluctuation has to be repeated at least three times), and the other three are the test periods. In the case of quarterly data it means $6 \cdot 4 = 24$ observed values. In the case of monthly data $6 \cdot 12 = 72$ months are appropriate. In the latter case the test period can start at $72/2 + 1$ that is at 37^{th} case. Analysing time series before extrapolation is necessary in order to discover the seasonality, trend, cycles and accidental changes. A longer time series provides chance of a better extrapolation.

Ones of the most popular methods are trend analysis and extrapolation. Trend analysis can discover permanent tendencies and trend-extrapolation is the projection of this tendency. A firm has complex functional processes; a basic tendency of these processes is the function of a lot of factors. A trend assumes a permanent effect in time, which is not always the case in practice. These differences can be significant. Sometimes it is a problem that last values of time series have greater influence on the future than previous ones, however a traditional trend-extrapolation ignores these facts. This problem can be solved by special procedures, e.g. exponential smoothing methods. Apart from trend analysis, discovery of periodical fluctuation and limitation of developmental conditions are important parts of extrapolation. It has to be stressed that using automatic trend-extrapolation is not correct; and it can rarely give proper extrapolation. Sometimes the application of other statistical or intuitive methods is more suitable. The longer the period of extrapolation is, the bigger the ratio of intuitive methods is according to some experts' opinions. Sometimes the latter one is the only acceptable method.

The traditional decomposition of time series are the following (*Makridakis* et *al*., 1983): trend, seasonality, cycles, and random changes. There may be a connection between them in additive or multiplicative ways. In the case of an additive connection, the model is:

$$X_t = T_t + S + C + E_t$$

where

X_t = time series observations (t =1, 2,...n),
T = trend,
S = seasonality,
C_t = business cycles (for instance length of period can be e.g. 3, 9, 27, 54 year),
E_t = residual, error term.

In the case of a multiplicative connection the model is as follows:

$$X_t = T_t \cdot S \cdot C \cdot E_t$$

Exponential smoothing method, moving average

Exponential smoothing is the improved version of the moving average. Traditional moving average use identical weights for all cases,[3] while exponential smoothing gives greater emphasis on most current data, but it still does not need deep mathematical - statistical knowledge. Additionally, it does not need long processing time from the computer either.

The basic model of the exponential smoothing is (*Makridakis* et *al*., 1983):

$$S_t = \alpha P + (1-\alpha)Q$$

where Q and P change by the type of trend and seasonality.

[3] There are moving average methods using different weights as well.

Pegels (1969) classified smoothing methods according to their seasonality and trend component.

This classification is shown in Table 1.

Table 1

Connections between seasonality and trend

Trend	Seasonality none	Seasonality additive	Seasonality multiplicative
None	$P_t=X_t$ $Q_t=S_{t-1}$	$P_t=X_t-C_{t-L}$ $Q_t=S_{t-1}$	$P_t=X_t / D_{t-L}$ $Q_t=S_{t-1}$
Additive	$P_t=X_t$ $Q_t=S_{t-1}+A_{t-1}$	$P_t=X_t-C_{t-L}$ $Q_t=S_{t-1}+A_{t-1}$	$P_t=X_t / D_{t-L}$ $Q_t=S_{t-1}+A_{t-1}$
Multiplicative	$P_t=X_t$ $Q_t=S_{t-1}\cdot B_{t-1}$	$P_t=X_t-C_{t-L}$ $Q_t=S_{t-1}\cdot B_{t-1}$	$P_t=X_t / D_{t-L}$ $Q_t=S_{t-1}\cdot B_{t-1}$

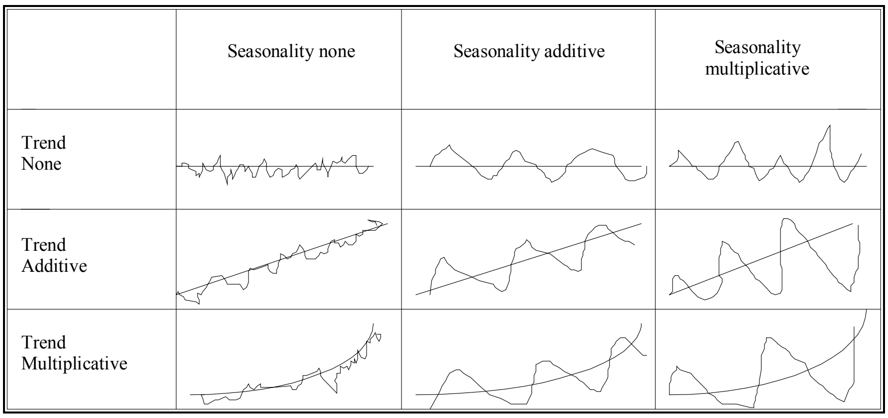
Source: *Makridakis* et *al*. (1983, p. 110).

where:

X_t = observed data,
S_t = smoothed data, $\alpha P+(1-\alpha)Q$,
$A_t = \beta(S_t - S_{t-1}) + (1-\beta)A_{t-1}$ (additive trend),
$B_t = \gamma(S_t/S_{t-1}) + (1-\gamma)B_{t-1}$ (multiplicative trend),
$C_t = \partial(X_t - S_t) + (1-\partial)C_{t-L}$ (additive seasonality),
$D_t = \theta(X_t/S_t) + (1-\theta)D_{t-L}$ (multiplicative seasonality),
L = length of seasonality.
Parameters $\alpha, \beta, \gamma, \partial, \theta$ are between 0 and 1.

Table 2 depicts equations of extrapolation (F_t+m) for different types of smoothing methods for m seasons.

Figure 1. Connections among the factors of time series



Source: *Makridakis* et *al*. (1983, p. 69)

Table 2

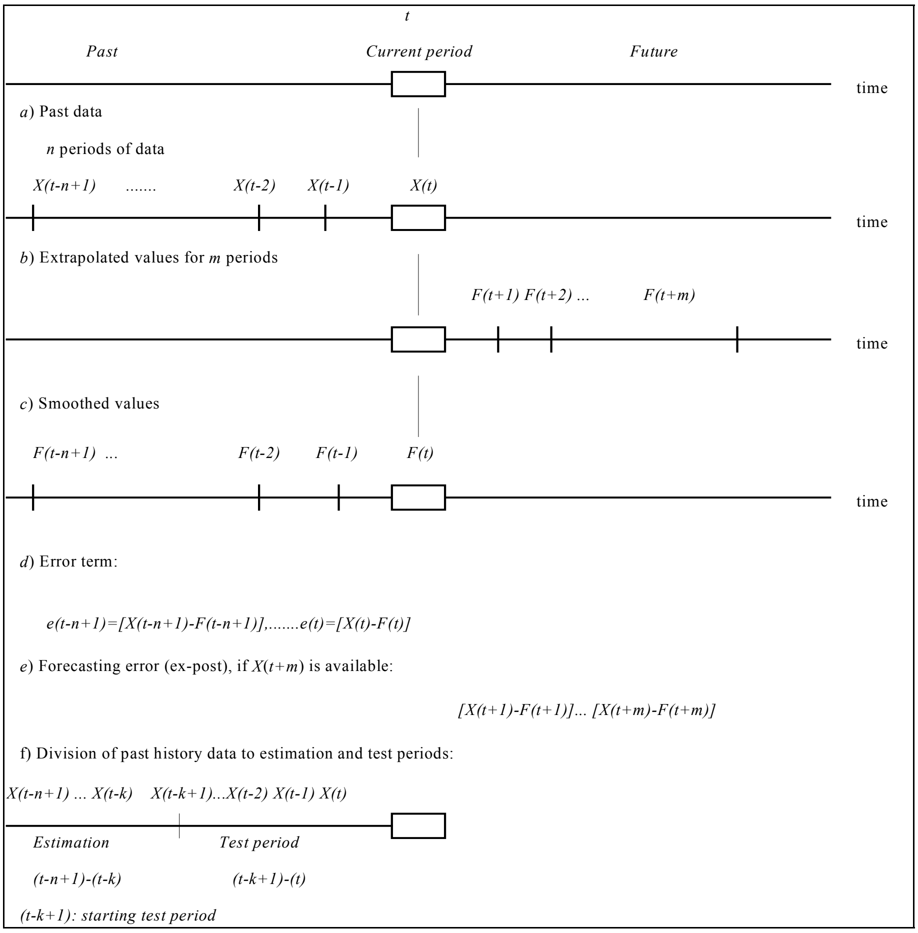
Equations of exponential smoothing methods

Trend	Seasonality		
	none	additive	multiplicative
None	S_t	$S_t + C_{t-L+m}$	$S_t \cdot D_{t-L+m}$
Additive	$S_t + mA_t$	$S_t + mA_t + C_{t-L+m}$	$(S_t + mA_t) \cdot D_{t-L+m}$
Multiplicative	$S_t \cdot B_t^m$	$S_t \cdot B_t^m + C_{t-L+m}$	$S_t \cdot D_{t-L+m} \cdot B_t^m$

Source: *Makridakis* et *al*. (1983, p. 111).

Let's assume that n periods of observation are available at the time of t, *and* the length of extrapolated period is m. Observed values are denoted by X, fitted values by F. In this case the correct meaning of elements can be seen in Figure 2.

Figure 2. Extrapolation procedure



Source: Part of the figure is based on: *Makridakis* et *al*. (1983, p. 66).

Algorithms of applied methods

Pegels' classification contains 9 methods, three additional methods are selected from Makridakis (1983). The first 9 rows of the following list comprise the combinations of the three seasonality types and the three trend components.

1 = Single exponential smoothing
2 = Seasonality - additive, trend none
3 = Seasonality - multiplicative, trend none
4 = Seasonality - none, trend additive (Holt's method)
5 = Seasonality - additive, trend additive
6 = Seasonality - multiplicative, trend additive (Winters' method);
7 = Seasonality - none, trend multiplicative
8 = Seasonality - additive, trend multiplicative
9 = Seasonality - multiplicative, trend multiplicative
10 = Adaptive Response Method (ARRSES)
11 = Brown one-parameter linear method
12 = Brown one-parameter quadratic method

T1 – Normal exponential smoothing:

$$F_{t+1} = \alpha X_t + (1 - \alpha) F_t$$

or (according to Table 1):

$$S_t = \alpha X_t + (1 - \alpha) S_{t-1}$$

Initialisation: given that F_1 is not known, the most frequent initialisation is: $F_1 = X_1$.
Extrapolation horizon is only 1 period: $F_{t+1} = S_{t+1}$

T2 – Seasonality additive, trend none:

$$S_t = \alpha (X_t - C_{t-L}) + (1 - \alpha) S_{t-1}$$
$$C_t = \partial (X_t - S_t) + (1 - \partial) C_{t-L}$$

where L = the length of a season, e.g. 4 in the case of quarterly data.

Initialisation: See Method 6 for the initialisation of C.
Extrapolation for m periods: $F_{t+m} = S_t + C_{t-L+m}$

T3 – Seasonality multiplicative, trend none:

$$S_t = \alpha (X_t / D_{t-L}) + (1 - \alpha) S_{t-1}$$
$$D_t = \theta (X_t / S_t) + (1 - \theta) D_{t-L}$$

Initialisation: See Method 6 for the initialisation of D (seasonal component).
Extrapolation for m periods: $F_{t+m} = S_t \cdot D_{t-L+m}$

T4 – Seasonality none, trend additive (Holt's method):

$$S_t = \alpha X_t + (1 - \alpha)(S_{t-1} + A_{t-1})$$
$$A_t = \beta (S_t - S_{t-1}) + (1-\beta) A_{t-1}$$

This procedure is identical with Holt's method, which applies the parameter of b_t, instead of A_t, and γ instead of β.

Holt's linear two parameter method: additive linear trend for t observed date with two parameters [α and γ]:

$$S_t = \alpha X_t + (1 - \alpha)(S_{t-1} + b_{t-L})$$
$$b_t = \gamma (S_t - S_{t-1}) + (1 - \gamma) b_{t-1}$$

Initialisation: S_t (initial value) and b_1 (trend) should be determined. S_1 can be equal to X_1. b_1 (trend component) can be determined by different ways. Two of them are:

$$b_1 = x_2 - x_1$$
$$b_1 = \frac{(x_2 - x_1) + (x_3 - x_2) + (x_4 - x_3)}{3}$$

Extrapolation for m periods: $F_{t+m} = S_t + mb_t$

T5 – Seasonality additive, trend additive:

$$S_t = \alpha (X_t - C_{t-L}) + (1 - \alpha)(S_{t-1} + A_{t-1})$$
$$A_t = \beta (S_t - S_{t-1}) + (1 - \beta) A_{t-1}$$
$$C_t = \partial (X_t - S_t) + (1 - \partial) C_{t-L}$$

Initialisation: See Method 6 for the initialisation of C, and Method 4 for the initialisation of the trend component (A_t).

Extrapolation for m periods: $F_{t+m} = S_t + mA_t + C_{t-L+m}$

T6 – Seasonality multiplicative, trend additive, Winters' three parameter trend and seasonality method:

This method comprises three smoothing methods. The overall smoothing equation is:

$$S_t = \alpha (X_t / (D_{t-L})) + (1 - \alpha)(S_{t-1} + A_{t-1})$$

Trend component:

$$A_t = \beta (S_t - S_{t-1}) + (1 - \beta)(A_{t-1})$$

Seasonal component:

$$D_t = \theta (X_t / S_t) + (1 - \theta)(D_{t-L})$$

Forecast: $F_{t+m} = (S_t + b_t m)\, I_{t-L+m}$

Initialisation: Let us assume that L=4 (quarterly data). In this case I_1 to I_4 should be estimated by means of the first four X values (I_1=X_1 / ((X_1+X_2+X_3+X_4)/4)), and b can be estimated as follows (*Makridakis* et *al*., 1983, p.108):

$$b = \frac{1}{L}\left[\frac{X_{L+1} - X_1}{L} + \frac{X_{L+2} - X_2}{L} + \ldots + \frac{X_{L+L} - X_L}{L}\right]$$

where it is convenient to use two complete seasons.

Extrapolation for m periods: $F_{t+m} = (S_t + A_t m)\, D_{t-L+m}$

This method is the same as Winters' method which applies parameter b_t for trend, and parameter I_t for seasonality:

$$S_t = \alpha\,(X_t / (I_{t-L})) + (1 - \alpha)(S_{t-1} + b_{t-1})$$
$$b_t = \gamma\,(S_t - S_{t-1}) + (1 - \gamma)(b_{t-1})$$
$$I_t = \beta\,(X_t \,/\, S_t) + (1 - \beta)(I_{t-L})$$
$$F_{t+m} = (S_t + b_t\, m)\; I_{t-L+m}$$

T7 – Seasonality none, trend multiplicative:

$$S_t = \alpha X_t + (1 - \alpha)(S_{t-1} \cdot B_{t-1})$$
$$B_t = \gamma\,(S_t \,/\, S_{t-1}) + (1 - \gamma)(B_{t-1})$$

Initialisation: See Method 4 for the initialisation of the trend component (B_t).

Extrapolation for m periods: $F_{t+m} = S_t B_t^m\; I_{t-L+m}$

T8 – Seasonality additive, trend multiplicative:

$$S_t = \alpha\,(X_t - C_{t-L}) + (1 - \alpha)(S_{t-1} \cdot B_{t-1})$$
$$B_t = \gamma\,(S_t \,/\, S_{t-1}) + (1 - \gamma)(B_{t-1})$$
$$C_t = \partial\;(X_t - S_t) + (1 - \partial)\; C_{t-L}$$

Initialisation: See Method 6 for the initialisation of C, and Method 4 for the initialisation of the trend component (B_t).

Extrapolation for m periods: $F_{t+m} = S_t B_t^m + C_{t-L+m}$

T9 – Seasonality multiplicative, trend multiplicative:

$$S_t = \alpha\,(X_t \,/\, D_{t-L}) + (1 - \alpha)(S_{t-1} \cdot B_{t-1})$$
$$B_t = \gamma\,(S_t \,/\, S_{t-1}) + (1 - \gamma)(B_{t-1})$$
$$D_t = \theta\,(X_t \,/\, S_t) + (1 - \theta)\; D_{t-L}$$

Initialisation: See Method 6 for the initialisation of D, and Method 4 for the initialisation of the trend component (B).

Extrapolation for m periods: $F_{t+m} = S_t D_{t-L} + m B_t^m$

Using the program of ExpS in the case of types 5, 6, 8 and 9 (where both the trend and seasonal components are calculated), the first parameter (/1) is the parameter of seasonality (additive: ∂, multiplicative: θ); the second (/2) is the parameter of the trend (additive: β, multiplicative: γ).

T10 – Adaptive Response Method (ARRSES):

The method will always change the parameter α_t automatically if the pattern changes in the time series, therefore the time-invariant α value will be replaced by the time-dependent α_t.

$$F_{t+1} = \alpha_t X_t + (1 - \alpha_t) F_t,$$

where

$\alpha_{t+1} = |E_t / M_t|$,
$E_t = \beta e_t + (1 - \beta)(E_{t-1})$,
$M_t = \beta |e_t| + (1 - \beta)(M_{t-1})$,
$e_t = X_t - F_t$,
α and β are between 0 and 1,
e_t is the error term,
E_t is the error term of smoothing and
M_t is the absolute error term of smoothing.

About the value of α_{t+1} the following note should be added: if forecasted values are good, then e_t will frequently change, therefore the numerator (E_t), together with α_{t+1} will be a small value. As a consequence, smoothed values get bigger weights, according to the original smoothing equation. However, if the sign of e_t does not change for a longer time, then the value of α_{t+1} will be higher, with a bigger weight of observed data.

Initialisation:

$F_2 = X_1$,
$\alpha_2 = \alpha_3 = \alpha_4 = \beta = 0.2$,
$E_1 = M_1 = 0$ and
β is a constant term that can control α_t.

The of order calculation is the following:

1. e_2;	2. E_2;	3. M_2	4. F_3;	
5. e_3;	6. E_3;	7. M_3	8. F_4;	
9. e_4;	10. E_4;	11. M_4	12. F_5;	
13.e_5;	14. E_5;	15. M_5	16. α_5;	17. F_6;
18.e_6;	19. E_6;	20. M_6	21. α_6;	22. F_7; etc.

Extrapolation: only 1 period ahead.

T11 – Brown one parameter linear method:

This method is a double exponential smoothing. The first smoothed values (S_t^1) will be smoothed again (S_t^2) because there is an assumed linear trend in the time series. Basically, it estimates a linear trend.

This method gives decreasing weights for past data:

$$S_t^1 = \alpha X_t + (1-\alpha)S_{t-1}^1,$$

$$S_t^2 = \alpha S_{t-1}^1 + (1-\alpha)S_{t-1}^2,$$

where

$$a_t = 2S_t^1 - S_t^2,$$

$$b_t = \frac{\alpha}{1-\alpha}\left(S_t^1 - S_t^2\right).$$

Initialisation: $S_1{}^1 = X_1$.

Extrapolation: $F_{t+m} = a_t + b_t m$

T12 – Brown one parameter quadratic method:

This method assumes a second order trend in the time series, therefore a third smoothing step is performed. Basically, it estimates a parabola of a second degree.

$$S_t^1 = \alpha X_t + (1-\alpha)S_{t-1}^1,$$

$$S_t^2 = \alpha S_t^1 + (1-\alpha)S_{t-1}^2,$$

$$S_t^3 = \alpha S_t^2 + (1-\alpha)S_{t-1}^3,$$

where

$$a_t = 3S_t^1 - 3S_t^2 + S_t^3,$$

$$b_t = \frac{\alpha}{2(1-\alpha)^2}\left[(6-5\alpha)S_t^1 - (10-8\alpha)S_t^2 + (4-3\alpha)S_t^3\right]$$

$$c_t = \frac{\alpha^2}{(1-\alpha)^2}\left(S_t^1 - 2S_t^2 + S_t^3\right)$$

Initialisation: $S_1^1 = S_1^2 = S_1^3 = X_1$

Extrapolation for *m* periods: $F_{t+m} = a_t + b_t m + \frac{1}{2}c_{t-L+m}$

Univariate statistics[4]

This exponential smoothing program uses different statistics of the error terms in order to measure the ‘goodness of fit’. Four of them participate in the model building process of the program: *MAE*, *SDE*, Durbin–Watson statistic and Theil’s *U* statistic (see their role in the overview of the program in the next section). For other statistics only the calculation method will be described here.

a) *ME* – Mean Error

$$ME = \sum_{i=1}^{n} e_i / n,$$
$$e_i = X_i - F_i,$$

where

F_i is the smoothed value,
X_i is the observed values.

The problem of this statistic is that positive and negative error terms equalise each other therefore further statistic, *MAE*, *SSE* and *MSE* were created to eliminate this problem.

b) *MAE* – Mean Absolute Error

MAE is the average of absolute values of the error terms. The less the value is, the closer the smoothed values are to the observed ones.

$$MAE = \sum_{i=1}^{n} |e_i| / n$$

c) *SSE* – Sum of Squared Errors

$$SSE = \sum_{i=1}^{n} e_i^2$$

d) *MSE* – Mean Squared Error

$$MSE = \sum_{i=1}^{n} e_i^2 / n$$

e) *SDE* – Standard Deviation of Errors

$$SDE = \sqrt{\sum_{i=1}^{n} e_i^2 / (n-1)}$$

[4] Statistics, used in ExpS for Windows are described here on the basis of *Makridakis* et *al*., (1983).

f) PE_i – Percentage Error

$$PE_i = \frac{X_i - F_i}{X_i}(100)$$

g) *MPE* – Mean Percentage Error

$$MPE = \sum_{i=1}^{n} PE_i / n$$

h) *MAPE* – Mean Absolute Percentage Error

$$MAPE = \sum_{i=1}^{n} \left| PE_i \right| / n$$

i) Theil's *U* statistic

$$U = \sqrt{\frac{\sum_{i=1}^{n-1}\left(\frac{F_{i+1} - X_{i+1}}{X_i}\right)^2}{\sum_{i=1}^{n-1}\left(\frac{X_{i+1} - X_i}{X_i}\right)^2}}$$

This is the most important statistic in the program. This value is calculated at each iteration, and the selection of the smoothing type and parameter set is based on the minimum value of Theil's *U* statistic. The closer the smoothed value is to the observed value, the smaller the nominator of is. This value is close to zero, if a good smoothing model has been applied. If the value is bigger than 1, then it is better to replace F_{t+1} with X_t, because this 'naiv' method provides a better extrapolation as a whole in the case of the simple exponential smoothing. If trend or seasonal component is calculated, then forecasted values will be adjusted accordingly with these parameters, therefore it can provide a better forecasting than the value of the previous period.

Another (similar) measure of evaluation is *MBA*.

j) *MBA* – McLaughlin Batting Averages

$$MBA = [4 - U] \cdot 100$$

k) *DW* – Durbin–Watson statistic

If F_i smoothed values comprise all important factors (trend, seasonality, cycles) then e_i-s are expected to be free of autocorrelation. *DW* statistic is one way to test the first order autocorrelation. This value is between 0 and 4 with an expected value of 2. The closer the value to 2 is, the more random the change and size of the error terms are, therefore, the better the chance is that subsequent error terms are not correlated with each other.

$$d = \frac{\sum_{i=2}^{n}\left(e_i - e_{i-1}\right)^2}{\sum_{i=1}^{n} e_i^2}$$

2. ExpS for Windows – a computer program

ExpS for Windows allows the automatic selection of the smoothing type, and additionally, it calculates a 'what if' and a sensitivity analysis.

The program starts with the main screen, shown in Figure 3. Users can set different parameters manually, or can ask for their automatic calculation. Initial parameters, smoothing parameters, length of seasonality, trend and seasonal parameters can either be set manually, or be calculated by the program.

In the case of a large computer speed or a lot of available time, users can set all parameters for automatic calculation. However, it is reasonable to ask for some parameters as automatic ones, while others have to be set manually. It is a good way to set manually the *initial value* for F_1 (to X_1), and the *length of seasonality* (to the theoretical value, such as 4 in the case of quarterly data). This procedure is used in the application part of this paper.

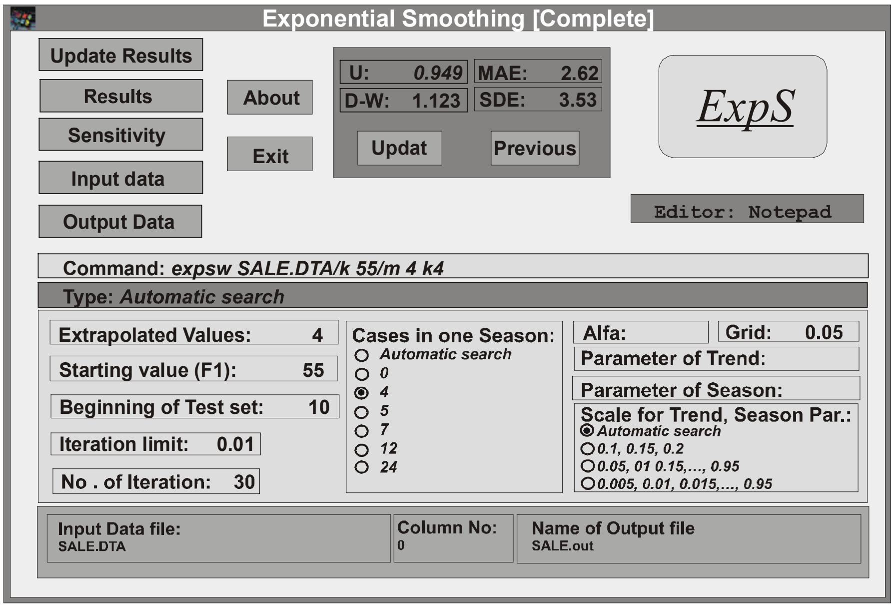
Trend and seasonal parameters between 0 and 0.2 are frequently used in practice (*Makridakis* et *al*., 1983), therefore, the first option is the 'Scale of Trend, Seasonal par.' section comprises only these values. In later stages user can ask for more subtle calculation. However, the most important step is at the first stage to set the 'type' to 'automatic search' as it happens to be in this example in the main screen. The program scans all the possibilities and provides one case from each type in order to compare different types. The best type is selected automatically, and details of the best model are described just after the summary table. The basic tool of the selection is the Theil's U statistic, discussed before.

After this automatic selection a summary table is provided (see Table 4 in the application section). The applied method will always assume that the recognised trend or seasonality is stable in the time series. If for example the ninth method was the best, then stable multiplicative trend and seasonality would be assumed. If this pattern changes during the test period, then the extrapolation will be uncertain. In such a case, the stability of time series has to be checked. There is a built in sensitivity analysis to check this factor. The program has a parameter of 'beginning of test set'. It means that the residuals are calculated only after this period to the end of the time series. If the seasonality and trend are stable in the time series, then different 'beginning of the test set' will provide similar results, similar to U statistics. This program's sensitivity analysis sets three different test periods and calculates the appropriate U statistics. Obviously, all the other parameters and the types of smoothing are unchanged. These U values can be applied to test the *stability* of the model, since the model can be considered as a stable one if U values are close to each other. There is no exact measurement for the size of this type of variation, it is only an experimental value. In case of stable data, the selected method can probably be applied effectively for extrapolating purposes. If the time series is not stable, the extrapolation can be uncertain. In this case the method of CENSUS II (*Herman–Kiss*, 1987) can be applied which is appropriate for managing the changing seasonality and trend, in the case of monthly data. In ExpS these starting periods are the half, two thirds and four fifths of the time series, respectively. In the case of quarterly data and a six-year time series (6·4 = 24 observations) these data are 13, 19 and 21 respectively. In the empirical part of this paper, we have 81 monthly data, where these starting periods are 41, 54, 64 (see in Section 4).

After the selection of the best parameter set, a 'What if' analysis is performed within the program (see Figure 4 in the application section). We explain the 'What if' analysis in the following example. Let us assume that we have quarterly data, and we are interested in the results of this model in order to compare them to actual data. Obviously, it is impossible to compare them to actual future values; therefore we can only use our own last years' four observations as actual data. A reasonable way is to compare the estimated results with actual data, if we assume that we have known this model for a year, and performed an extrapolation. This is the so called 'what if' analysis: 'What would have happened, if we had known this model earlier?' This model provides extrapolated values, denoted by '=', with these optimal parameters and values. The observed values can be found in the last column. Comparing observed values to fitted ones, the decision maker can assess the reliability of the given model. It is not impossible that a year earlier we had a different parameter set for the shorter time series, however, one solution had to be selected.

The main screen of the program is depicted in Figure 3. Data on the main screen are not relevant now; displaying the structure of the program is the only purpose of this screen.

Figure 3. Main Screen of ExpS



Clicking the *Update Results* (RUN) button will result in the actual running of the ExpS program with the preset parameters. The *Results* button will show the results of the model.

Sensitivity analysis can help a faster model building (see the explanation before). With the help of the following parameters, the user can build up an arbitrary model.

Starting value (initial value) (F_1) is of decisive importance in the case of each exponential smoothing method. They can frequently change the results to a great extent in either a positive or a negative direction. Estimation of a good quality initial value is essential. Iteration of the initial value is the following: the program generates five different initial values, and the best value is selected (where the U statistic is the smallest one). The five possible initial values are the following:

– minimum,
– maximum,
– mean,
– mean-minimum/2,
– minimum+(maximum-minimum)/2

computed from the first part of the time series in study.

The '*Season*' field will set the number of periods in one season. If there is no seasonality, it can be set to *zero*. In the case of *Automatic search* the program finds the best of the preset period-lengths that may be 4, 5, 7, 12 or 24.

There is a command line in the middle of the screen, which will be used at the updating process.

Apart from the methodological uniqueness, some other special features, user friendly solutions exist within the program.

1. Users can build up their own model in one screen by updating (perhaps) the four most important statistics, the U statistics, the DW statistics, the MAE and the SDE (for the error terms; see Section 1). The '*Update*' and '*Previous*' buttons will always switch the current and the previous results to follow the changes. If the user has run a different model previously, then the '*Previous*' button will show the statistics of that model.

2. In the case of an uncertain structure, an automatic selection of the length of the seasonality is allowed (see the middle of the main screen).

3. The user can see the observed, forecasted values and the error term in one common chart (see Figure 4 in the application section).

3. Comparison with other program packages

SPSS for Windows[5] 7.5 and Statistica for Windows[6] 4.3 are the bases of this comparison.

All the programs allow for selecting both the trend and seasonal components separately, which means an arbitrary combination of theirs. ExpS uses *Makridakis*' suggestion for the trend component: the choice among none / additive / multiplicative

[5] SPSS Inc. 1996.
[6] StatSoft Inc. 1993.

types of trend (see the theoretical part) is offered. SPSS and Statistica uses four different types of trends: none / linear / exponential / dumped. All the programs use none / additive / multiplicative type seasonal components.

All the programs are able to perform the automatic calculation of the initial values of; the smoothing (α), the trend (γ); and the seasonal parameters (δ). The methodology of the calculation of the initial value is not known in SPSS and Statistica. Table 3 depicts the main features of the mentioned program packages.

Table 3

Comparison of different exponential smoothing programs

Features	ExpS	SPSS	Statistica
Automatic selection of smoothing parameter	X	X	X
Automatic trend parameter selection	X	X	X
Automatic seasonal parameter selection	X	X	X
Automatic initial value calculation	X	X	X
Automatic selection of seasonality	X		
Automatic selection of the method	X		
Comparison of different methods	X		
Sensitivity analysis	X		
What if analysis	X		
ACF* calculation			X

* Autocorrelation function.

SPSS and Statistica are complex program packages, therefore they allow using the smoothed values as a separate variable. ExpS allows saving data in case one would like to use the original and fitted values of the best model. There is a special data file with '.ft1' extension. These data can be used for other statistical (e.g. SPSS, BMDP), graphical (Harvard Graphics), Spreadsheet (Excel, Lotus), or Database (Paradox, Dbase) programs, because it is an ASCII text file.

4. Application – The number of visitors in Hungary

Data are collected about the number of visitors in Hungary (from European countries), from January 1992 to September 1998[7] that means 81 monthly data.[8] Observation for 1988 can be followed on Figure 4 ('X-i' column). ExpS for Windows prepared a summary table about the results of the twelve methods that is shown in Table 4, where the explanation of columns:

T – type, smoothing method,
Alpha – value of α,
$p1, p2$ – the first (trend) and second (seasonal) parameters, if any,
L – length of one season, if any.

[7] Source: Statisztikai Havi Közlemények. Központi Statisztikai Hivatal, Budapest.
[8] Data are available from the authors: kisst@ktk.jpte.hu

The other columns displayed are explained in Section 1.

Table 4

Summary table of different methods about the number of visitors in Hungary

T	Alpha	*p*1	*p*2	*L*	*ME*	*MAE*	*MAPE*	*SDE*	*MSE*	*DW*	*U*	*MBA*
1	1.050	0.000	0.000	0	-5.7	615.3	19.0	850.7	706003	1.951	0.993	301
2	**0.350**	**0.100**	**0.000**	**12**	**-54.7**	**192.9**	**6.1**	**253.2**	**62533**	**1.414**	**0.384**	**362**
3	1.050	0.100	0.000	12	848.3	930.0	25.8	1278.1	1593604	0.553	1.518	248
4	1.050	0.100	0.000	0	-5.8	636.5	20.0	890.8	774097	1.959	1.036	296
5	0.750	0.100	0.100	12	-24.7	204.2	6.7	262.5	67210	2.043	0.406	359
6	0.450	0.200	0.100	12	1026.9	1046.5	28.9	1370.2	1831564	0.393	1.660	234
7	1.050	0.100	0.000	0	-112.2	639.5	20.2	926.6	837695	1.907	1.029	297
8	0.750	0.100	0.100	12	-32.2	207.3	6.8	263.6	67801	2.071	0.404	360
9	0.450	0.200	0.100	12	1013.5	1035.0	28.5	1359.7	1803582	0.399	1.646	235
10	0.150	0.100	0.000	0	91.9	697.3	22.7	963.8	906209	0.828	1.378	262
11	0.650	0.000	0.000	0	-30.4	673.8	21.8	986.2	948922	1.918	1.133	287
12	0.350	0.000	0.000	0	-17.9	780.1	25.4	1067.6	1111892	1.416	1.361	264

Table 4 shows that method 2 – 'additive seasonality – no trend' – has the smallest *U* value: 0.384. The parameter set of this model is further refined. Method 2 was set at the 'type' section of the program, and we have searched for a better parameter set. The final *U* value was 0.379. The smoothing parameter α and the seasonal parameter equally have a final value of 0.4. Initial value was 2398. The sensitivity analysis of this result can be seen in as follows:

Alpha : 0.4000
Beginning of Test: 41 U: 0.3791
Beginning of Test: 54 U: 0.3063
Beginning of Test: 64 U: 0.2993

Ratio (*R*) of the smallest and biggest *U* values denotes 21 percent difference.

$$R = \frac{|U_{\min} - U_{\max}|}{U_{\max}} = \frac{|0.2993 - 0.3791|}{0.3791} = 0.21 .$$

(In the case of absolute stability – with the same *U* values – the difference is 0 percent).

The difference seems to be big enough to reject the model. However, the second and the third *U* statistics are very close to each other, and additionally, these data are closer to the current date. Consequently, this analysis can be accepted. A decision maker should have a look at the 'What if' analysis as well, to study the behaviour of the 'quasi' forecast. Figure 4 depicts the 'What if' analysis of the model.

These forecasts are sometimes rather accurate. Comparing the values of observed (*X*-i,'+') and 'what if' (Whatif '=') values it can be seen they are very close to each other, apart from the latest three periods (the last quarter of the year). As a consequence, decision makers can accept this model and can use the forecasts in the future. However, forecasts for the last quarter need special attention.

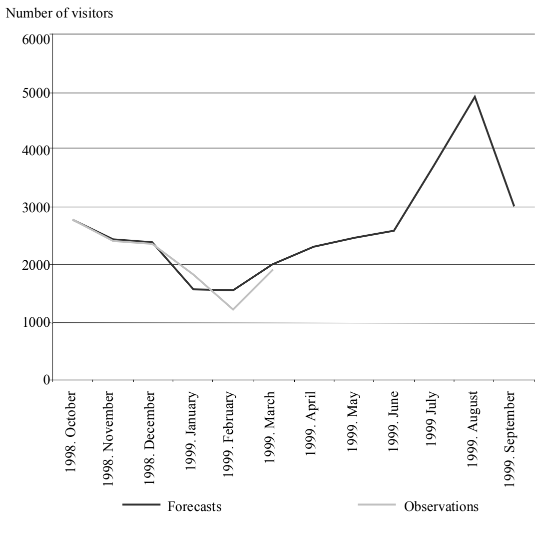
Figure 4. What if analysis

```
Type ( 2)  : Seasonality - additive, Trend none

 Time X-i (+)  F-i (-) WhatIf (=) -> Fitted values             Error    WhatIf
  72  2661.4  2551.8 ł          -=                       ł   109.64( 2662.06)
  73  1866.2  1772.2 ł    -=                             ł    94.02( 1838.62)
  74  1816.1  1807.9 ł     =                             ł     8.22( 1836.71)
  75  2193.2  2306.0 ł        +=                         ł  -112.76( 2331.50)
  76  2562.3  2512.4 ł           =                      ł    49.85( 2583.10)
  77  2612.6  2733.5 ł            +=                     ł  -120.87( 2784.18)
  78  2852.5  2763.3 ł              -=                   ł    89.17( 2862.38)
  79  3610.2  4019.6 ł                     +   =         ł  -409.35( 4082.94)
  80  4854.7  4997.2 ł                              +- = ł  -142.54( 5224.37)
  81  3000.0  3025.0 ł                *   =            ł   -25.03( 3309.17)
-----------------------------------------------------------------------------------
  82          2746.3 ł              -                    ł
  83          2435.1 ł          -                        ł
  84          2394.2 ł          -                        ł
  85          1567.0 ł   -                               ł
  86          1544.5 ł  -                                ł
  87          2010.3 ł      -                            ł
  88          2300.9 ł         *                         ł
  89          2461.0 ł          -                        ł
  90          2589.6 ł           -                       ł
  91          3690.5 ł                     -             ł
  92          4896.0 ł                              -    ł
```

The last figure of this paper shows the line-diagrams of the observations and the line-diagrams of the observations and the forecasts.

Figure 5. Comparison of forecasts and observations



As a *summary*, it is reasonable to say that ExpS for Windows is a useful tool in extrapolating the number of visitors in Hungary. The automatic type selection helped to

choose the best method; the sensitivity analysis provided a deeper insight into the stability of the model; and finally the 'what if' analysis helped us to evaluate the behaviour of the time series in order to decide whether to accept the results or not.

REFERENCES

AERTS, M. – AUGUSTYNS, I. – JANSSEN, P. (1997): Smoothing sparse multinomial data using local polynomial fitting. *Journal of Nonparametric Statistics*, **8**, pp. 127–147.

CLEVELAND, W. S. – LOADER, C. (1996): Smoothing by local regression: principles and methods (with discussion). In Statistical theory and computational aspects of smoothing. (Eds.: Hardle, W. – Schimek, M.G.) *Physica-Verlag*, Heidelberg, pp. 10–49; 80–102; 113–120.

EFRON, B. – TIBSHIRANI, R. (1996): Using specially designed exponential families for density estimation. *Annals of Statistics*, **24**, pp. 2431–2461.

EILERS, P. H. C. – MARX, B. D. (1996): Flexible smoothing with B-splines and penalties (with discussion). *Statistical Science*, **11**, pp. 89–121.

FAN, J. – HALL, P. – MARTIN, M. – PATIL, P. (1996): On local smoothing of nonparametric curve estimators. *Journal of the American Statistical Association*, **91**, pp. 258–266.

GLEICK, J. (1987): Chaos – making a new science, *Penguin Books Ltd*, Harmondsworth, Middlesex.

HARDLE, W. – MARRON, J. S. (1995): Fast and simple scatterplot smoothing. *Computational Statistics and Data Analysis*, **20**, pp. 1–17.

JONES, M. C. – FOSTER, P. J. (1996): A simple nonnegative boundary correction method for kernel density estimation. *Statistica Sinica*, **6**, pp. 1005–1013.

MAKRIDAKIS, S. – WHEELWRIGHT, S. C. – MCGEE, V. E. (1983): Forecasting methods for management. *John Wiley & Sons*, New York – Chichester – Brisbane – Toronto – Singapore.

PERCIVAL, D. – MOFJELD, H. (1997): Analysis of subtidal coastal sea level fluctiations use in wavelets. *Journal of American Statistical Association*, **92**.

WAHBA, G. – WANG, Y. – GU, C. – KLEIN, R. – KLEIN, B. (1995): Smoothing spline ANOVA for exponential families, with application to the Wisconsin epidemiological study of diabetic retinopathy. *Annals of Statistics*, **23**, pp. 1865–1895.