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Comparing different ranking functions for solving fuzzy linear programming problems with fuzzy cost coefficients*

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In many applications of linear programming, the lack of exact information results in various problems. Nevertheless, these types of problems can be handled using fuzzy linear programming. This study aims to compare different ranking functions for solving fuzzy linear programming problems in which the coefficients of the objective function (the cost vector) are fuzzy numbers. A numerical example is introduced from the field of tourism and then solved using five ranking functions. Computations were carried out using the FuzzyLP package implemented in the statistical software R.

KEYWORDS: fuzzy linear programming, fuzzy sets, ranking functions

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Linear programming (LP) is one of the most widely used mathematical techniques for solving real-world problems in which the goal is to maximize (or minimize) a linear objective function subject to a finite number of linear constraints. These problems are characterized by a panoply of possible solutions that satisfy a set of linear constraints. The selection of the optimal solution depends on the specific goal of the problem defined in the objective (or cost) function. A large body of literature has illustrated its applicability, such as in the agricultural sector (*Alotaibi–Nadeem* [2021]), production processes (*Krynke–Mielczarek* [2018]), the field of energy (*Polat–Gürtuna* [2018]), and transportation (*Patel–Patel–Bhathawala* [2017]).

Scholars have placed a special interest in developing a flexible approach for handling imprecise, inaccurate, or vague information presented in LP problems. Fuzzy linear programming has emerged and gained significant interest (*Ghanbari et al.* [2020], *Saati et al.* [2015], *Cardenas–Verdegay* [1997]). Following this line of research, we aim to provide a comparative analysis for solving fuzzy linear programming problems with fuzzy cost coefficients using different ranking functions.

The study is organised as follows: after having given an overview of the classical LP problem in Section 1, Section 2 provides an introduction of fuzzy theory. Section 3 discusses fuzzy linear programming and Section 4 then addresses ranking functions. Next, Section 5 examines a numerical example from the field of tourism. The work ends with a conclusion in Section 6, summarising the main findings and giving recommendations to practitioners.

1. Classical LP problem

The classical (or crisp) LP problem aims to maximize (or minimize) a linear objective function subject to a finite number of linear constraints. Without loss of generality, the general LP problem can be formulated as follows (*Nasseri–Ebrahimnejad–Cao* [2019]):

$$\begin{aligned} \max \mathbf{z} &= \mathbf{c}\mathbf{x} && (1) \\ \text{s.t. } \mathbf{A}\mathbf{x} &\leq \mathbf{b} \\ \mathbf{x} &\geq \mathbf{0} \end{aligned}$$

where $\mathbf{c}^T = (c_1, \dots, c_n)^T \in \mathbb{R}^n$ is the cost vector, $\mathbf{x} = (x_1, \dots, x_n)^T \in \mathbb{R}^n$ is the vector of decision variables, $\mathbf{b} = (b_1, \dots, b_m)^T \in \mathbb{R}^m$ is the vector of coefficients (right-hand side of the constraints), and $\mathbf{A} = [a_{ij}]_{m \times n} \in \mathbb{R}^{m \times n}$ is the coefficient matrix (left-hand side of the constraints).

In this context, given a cost vector $\mathbf{c}^T = (c_1, \dots, c_n)^T$, the goal is to maximize (or minimize) the linear objective function $\mathbf{c}\mathbf{x} = \sum_{i=1}^n c_i x_i$ subject to a finite number of

linear constraints. A relevant disadvantage attached to this approach is that it assumes the researcher to have an exact knowledge of the model under consideration – the exact coefficients of the cost vector (\mathbf{c}), left-hand side of the constraints (\mathbf{A}), and right-hand side of the constraints (\mathbf{b}). It also assumes no ambiguity (or uncertainty) in the vector of decision variables, \mathbf{x} .

However, in many real applications, obtaining the required information may involve a considerable amount of time and effort. Indeed, it is not always possible to obtain the exact information but instead information with a high degree of imprecision (or fuzziness). Fuzzy theory provides a mathematical framework to model quantities that are imprecise, inaccurate, or vague. The combination of fuzzy theory and LP has emerged under the name ‘fuzzy linear programming’ (FLP) (Zimmermann [1978], Bector-Chandra [2005], Nasser-Ebrahimnejad-Cao [2019], Fang et al. [1999]). Thus, FLP provides a mathematical framework for approaching imprecise (fuzzy) information presented in many LP applications.

2. Preliminaries: fuzzy theory

Definition 1 (*Fuzzy set and membership function*). Let X be the universe whose generic element is denoted by x . A fuzzy set \tilde{C} is a function, $\tilde{C}: X \rightarrow [0, 1]$ that can be represented by a set of ordered pairs:

$$\tilde{C} = \left\{ (x, \mu_{\tilde{C}}(x)) \mid x \in X \right\}$$

where $\mu_{\tilde{C}}(\cdot)$ is called the membership function of the fuzzy set.

The membership function represents the strength with which x is contained in \tilde{C} . Intuitively, it can be interpreted as follows:

- If the membership degree of x is equal to 1, then value x definitely belongs to the fuzzy set.
- If the membership degree of x is equal to 0, then value x definitely does not belong to the fuzzy set.
- If the membership function belongs to $[0, 1]$, then it can be defined in several ways (e.g. representing a trapezoidal or triangular form).

In contrast, the membership function of the classical LP approach (see Section 1) can be defined using only two elements:

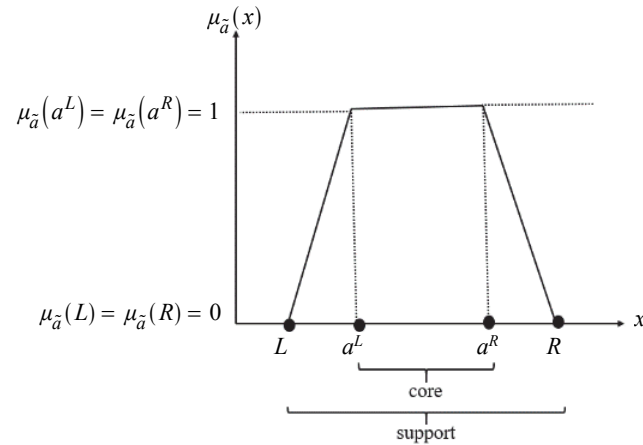
$$\mu_c(x) = \begin{cases} 0, & \text{if } x \in C \\ 1, & \text{if } x \notin C \end{cases}.$$

Definition 2 (*Support and core of a fuzzy set*). The support of fuzzy set \tilde{C} is the set of all $x \in X$ with $\mu_{\tilde{C}}(x) > 0$. The core of fuzzy set \tilde{C} is the set of all $x \in X$ with $\mu_{\tilde{C}}(x) = 1$.

Definition 3 (*Trapezoidal and triangular fuzzy numbers*). Let $\tilde{a} = (a^L, a^R, L, R)$ with $L < a^L < a^R < R$ being real numbers. Fuzzy number \tilde{a} is called trapezoidal fuzzy number (TrFN) if its membership function is:

$$\mu_{\tilde{a}}(x) = \begin{cases} \frac{x-L}{a^L-L} & \text{if } L \leq x \leq a^L \\ 1 & \text{if } a^L \leq x \leq a^R \\ \frac{R-x}{R-a^R} & \text{if } a^R \leq x \leq R \\ 0 & \text{otherwise} \end{cases}.$$

Figure 1. Graphical representation of a TrFN



A TrFN holds the following properties:

- $\mu_{\tilde{a}}(x)$ is non-decreasing on range $[L, a^L]$.
- $\mu_{\tilde{a}}(x)$ is non-increasing on range $[a^R, R]$.
- The support is defined in the interval $[L, R]$.
- The core is defined in the interval $[a^L, a^R]$.
- $\mu_{\tilde{a}}(x) = 0$ for all $x \in (-\infty, L]$ and all $x \in [R, \infty)$.

Particularly, if $a^L = a^R = a$, we obtain a triangular fuzzy number (TFN) denoted as $\tilde{a} = (a, L, R)$.

3. Fuzzy linear programming

Uncertainty (or fuzziness) in LP problems may occur in different ways. Depending on the source of this uncertainty, FLP problems can be classified in different groups. (See Table 1.)

In the present work, we focus on the solution for the first group (FLP problems with fuzzy cost coefficients) using ranking functions. The following section introduces several ranking functions.

Table 1

<i>Classification of FLP problems</i>	
Group	Mathematical model
<i>Group 1.</i> FLP problems with fuzzy cost coefficients, $\tilde{\mathbf{c}}$.	$\begin{aligned} \max \tilde{\mathbf{z}} &\approx \tilde{\mathbf{c}}\mathbf{x} \\ \text{s.t. } \mathbf{A}\mathbf{x} &\leq \mathbf{b} \\ \mathbf{x} &\geq \mathbf{0} \end{aligned}$
<i>Group 2.</i> FLP problems with fuzzy numbers for the coefficients of the decision variables in the constraints and the right-hand side of the constraints, $\tilde{\mathbf{A}}$ and $\tilde{\mathbf{b}}$, respectively.	$\begin{aligned} \max \mathbf{z} &= \mathbf{c}\mathbf{x} \\ \text{s.t. } \tilde{\mathbf{A}}\mathbf{x} &\leq \tilde{\mathbf{b}} \\ \mathbf{x} &\geq \mathbf{0} \end{aligned}$
<i>Group 3.</i> FLP problems with fuzzy cost coefficients, $\tilde{\mathbf{c}}$. Fuzzy numbers for the coefficients of the decision variables in the constraints and the right-hand side of the constraints, $\tilde{\mathbf{A}}$ and $\tilde{\mathbf{b}}$, respectively.	$\begin{aligned} \max \tilde{\mathbf{z}} &\approx \tilde{\mathbf{c}}\mathbf{x} \\ \text{s.t. } \tilde{\mathbf{A}}\mathbf{x} &\leq \tilde{\mathbf{b}} \\ \mathbf{x} &\geq \mathbf{0} \end{aligned}$
<i>Group 4.</i> FLP problems with fuzzy numbers for the decision variables, $\tilde{\mathbf{x}}$. Fuzzy numbers for the coefficients of the right-hand side of the constraints, $\tilde{\mathbf{b}}$.	$\begin{aligned} \max \tilde{\mathbf{z}} &\approx \mathbf{c}\tilde{\mathbf{x}} \\ \text{s.t. } \mathbf{A}\tilde{\mathbf{x}} &\leq \tilde{\mathbf{b}} \\ \tilde{\mathbf{x}} &\geq \mathbf{0} \end{aligned}$
<i>Group 5.</i> FLP problems with fuzzy cost coefficients, $\tilde{\mathbf{c}}$. Fuzzy numbers for the decision variables, $\tilde{\mathbf{x}}$. Fuzzy numbers for the coefficients of the right-hand side of the constraints, $\tilde{\mathbf{b}}$.	$\begin{aligned} \max \tilde{\mathbf{z}} &\approx \tilde{\mathbf{c}}\tilde{\mathbf{x}} \\ \text{s.t. } \mathbf{A}\tilde{\mathbf{x}} &\leq \tilde{\mathbf{b}} \\ \tilde{\mathbf{x}} &\geq \mathbf{0} \end{aligned}$
<i>Group 6.</i> FLP problems with fuzzy numbers for the decision variables, $\tilde{\mathbf{x}}$. Fuzzy numbers for the coefficients of the decision variables in the constraints and the right-hand side of the constraints, $\tilde{\mathbf{A}}$ and $\tilde{\mathbf{b}}$, respectively.	$\begin{aligned} \max \tilde{\mathbf{z}} &\approx \tilde{\mathbf{c}}\tilde{\mathbf{x}} \\ \text{s.t. } \tilde{\mathbf{A}}\tilde{\mathbf{x}} &\leq \tilde{\mathbf{b}} \\ \tilde{\mathbf{x}} &\geq \mathbf{0} \end{aligned}$
<i>Group 7.</i> FLP is fully fuzzy, involving fuzzy cost coefficients, $\tilde{\mathbf{c}}$. Fuzzy numbers in the decision variables, $\tilde{\mathbf{x}}$. Fuzzy numbers for the coefficients of the decision variables in the constraints and the right-hand side of the constraints, $\tilde{\mathbf{A}}$ and $\tilde{\mathbf{b}}$, respectively.	$\begin{aligned} \max \tilde{\mathbf{z}} &\approx \tilde{\mathbf{c}}\tilde{\mathbf{x}} \\ \text{s.t. } \tilde{\mathbf{A}}\tilde{\mathbf{x}} &\leq \tilde{\mathbf{b}} \\ \tilde{\mathbf{x}} &\geq \mathbf{0} \end{aligned}$

Note. In this table, we consider the following fuzzy notations: $\tilde{\mathbf{c}}^T = (\tilde{c}_1, \dots, \tilde{c}_n)^T$, $\tilde{\mathbf{b}} = (\tilde{b}_1, \dots, \tilde{b}_m)^T$, $\tilde{\mathbf{x}} = (\tilde{x}_1, \dots, \tilde{x}_n)^T$, and $\tilde{\mathbf{A}} = [\tilde{a}_{ij}]_{m \times n}$, with \tilde{c}_j , \tilde{b}_i , \tilde{x}_j , \tilde{a}_{ij} being fuzzy numbers, for $i = 1, \dots, m$, $j = 1, \dots, n$.

Source: Saati et al. [2015].

4. Ranking functions

For the FLP problem with fuzzy cost coefficients (Group 1 of Table 1), the FLP problem can be written as follows:

$$\max \tilde{\mathbf{z}} \approx \tilde{\mathbf{c}}\mathbf{x} \quad (2)$$

$$\text{s.t. } \mathbf{Ax} \leq \mathbf{b}$$

$$\mathbf{x} \geq \mathbf{0}$$

where $\tilde{\mathbf{c}}^T = (\tilde{c}_1, \dots, \tilde{c}_n)^T$, and \tilde{c}_j being fuzzy numbers for $j = 1, \dots, n$.

The solution of (2) can be carried out by converting the original problem into its equivalent crisp problem and then using standard methods (*Nasseri–Ebrahimnejad–Cao* [2019]). The transformation of a fuzzy problem into a crisp problem can be achieved using the concept of ranking function. A ranking function maps each fuzzy number into the real line, \mathbb{R} ,

$$R = F(R) \rightarrow \mathbb{R}.$$

Ranking functions are especially useful for comparing fuzzy numbers. In general, comparing fuzzy numbers can be extremely difficult (*McCahon–Lee* [1990]). If \tilde{a} and \tilde{b} are two fuzzy numbers, then both numbers can be compared using the following:

$$\tilde{a} > \tilde{b} \Leftrightarrow R(\tilde{a}) > R(\tilde{b})$$

$$\tilde{a} < \tilde{b} \Leftrightarrow R(\tilde{a}) < R(\tilde{b})$$

$$\tilde{a} \approx \tilde{b} \Leftrightarrow R(\tilde{a}) = R(\tilde{b})$$

where R denotes a ranking function.

Moreover, if the ranking function satisfies the condition $R(k \cdot \tilde{a} + \tilde{b}) = k \cdot R(\tilde{a}) + R(\tilde{b})$ for any two fuzzy numbers \tilde{a} and \tilde{b} and any $k \in \mathbb{R}$, then it is called a linear ranking function. To solve problem (2), we can transform it into an equivalent crisp problem, replacing its fuzzy cost function with

$$\max \mathbf{z} = R(\tilde{c}_1)x_1 + \dots + R(\tilde{c}_n)x_n$$

where R denotes a ranking function.

Table 2

Five ranking functions proposed in the literature

No.	Author	Function's name	Ranking function for TrFN
1	Yager [1978]	Yager's F_1	$R(\tilde{a}) = \frac{1}{3} \cdot \frac{\left[(a^R)^2 - (a^L)^2 \right] + (R^2 - L^2) + (Ra^R - La^L)}{(a^R - a^L) + (R - L)}$
2	Yager [1978]	Yager's F_3	$R(\tilde{a}) = \frac{L + a^L + a^R + R}{4}$
3	Adamo [1980]	Adamo	$R(\tilde{a}) = R - \alpha \cdot (R - a^R) \text{ for fixed } \alpha \in [0, 1]$
4	Campos–González [1989]	Campos	$R(\tilde{a}) = a^L + \lambda \cdot \left[(a^R - a^L) + \frac{L + R}{3} \right] - \frac{L}{3}$ <p>The parameter $\lambda \in [0, 1]$ may be interpreted as an optimist/pessimist degree which must be selected by the decision maker. An optimist choice is indicated by values close to 1, while the pessimist choice is indicated by values close to 0.</p>
5	González [1990]	González	$R(\tilde{a}) = a^L - \frac{a^L - L}{t + 1} + \lambda \cdot \left[a^R - a^L + \frac{(R - L) - (a^R - a^L)}{t + 1} \right],$ $\lambda \in [0, 1], t \geq 0$ <p>λ has the same definition as in Campos's function.</p>

5. Numerical example: estimating the maximum revenue generated by tourists in Venice

This section considers a recent real-world problem introduced in the field of tourism (Bertocchi *et al.* [2020]). The goal is to maximize the economic benefit generated by tourists in the Italian city of Venice under several socio-environmental constraints. The original FLP problem was formulated as follows:

$$\begin{aligned} \max \tilde{\mathbf{z}} &\approx \tilde{\mathbf{c}}\mathbf{x} \\ \text{s.t. } \tilde{\mathbf{A}}\mathbf{x} &\leq \tilde{\mathbf{b}} \\ \mathbf{x} &\geq \mathbf{0} \end{aligned}$$

where $\mathbf{x} = (\text{TH}, \text{NTH}, \text{E})^T \in \mathbb{R}^3$ is the vector of decision variables representing the type of tourists: TH (tourist in hotels), NTH (tourist in other types of accommodations), and E (excursionist or day tourist).

The cost vector $\tilde{\mathbf{c}}^T = (\tilde{c}_1, \tilde{c}_2, \tilde{c}_3)^T$ represents the economic benefit per day generated by each type of tourist stated as uncertain. To capture the uncertainty in these values, we defined the cost vector using the fuzzy numbers $\tilde{c}_1 = (190, 210, 230)$, $\tilde{c}_2 = (140, 160, 180)$, and $\tilde{c}_3 = (45, 60, 80)$. The constraints reflect the most frequently used tourism subsystems (and their maximum capacity) in the historical centre of Venice. This information is represented by

$$\tilde{\mathbf{A}}\mathbf{x} = \begin{pmatrix} & \text{TH} & \text{NT} & \text{E} \\ a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ \tilde{a}_{31} & \tilde{a}_{32} & a_{33} \\ \tilde{a}_{41} & \tilde{a}_{42} & a_{43} \\ \tilde{a}_{51} & \tilde{a}_{52} & a_{53} \\ \tilde{a}_{61} & \tilde{a}_{62} & a_{63} \\ \tilde{a}_{71} & \tilde{a}_{72} & a_{73} \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \quad \text{and} \quad \tilde{\mathbf{b}} = \begin{pmatrix} \tilde{b}_1 \\ \tilde{b}_2 \\ \tilde{b}_3 \\ \tilde{b}_4 \\ \tilde{b}_5 \\ \tilde{b}_6 \\ \tilde{b}_7 \end{pmatrix}.$$

The *first constraint* corresponds to the number of beds in hotels (from one- to five-star hotels), where $(a_{11}, a_{12}, a_{13}) = (1, 0, 0)$ and \tilde{b}_1 is a fuzzy number representing the uncertainty in maximum capacity, defined as $\tilde{b}_1 = (13,000; 14,400; 15,500)$. The *second constraint* is the number of beds in non-hotels, where $(a_{21}, a_{22}, a_{23}) = (0, 1, 0)$, and, similarly, \tilde{b}_2 is a fuzzy number representing the uncertainty in maximum capacity, defined as $\tilde{b}_2 = (18,500; 20,400; 22,000)$. The *third constraint* refers to the capacity (number of seats) in restaurants, pizzerias, and dining rooms. The occupancy by type of tourist is indicated by the fuzzy numbers $(\tilde{a}_{31}, \tilde{a}_{32}, \tilde{a}_{33}) = [(0.675, 0.750, 0.825), (0.585, 0.650, 0.715), (0.180, 0.200, 0.220)]$, and maximum capacity is shown by the fuzzy number $\tilde{b}_3 = (23,300; 24,000; 24,700)$. The *fourth constraint* is related to the number of available parking spaces. The occupancy by type of tourist is specified by the fuzzy numbers $(\tilde{a}_{41}, \tilde{a}_{42}, \tilde{a}_{43}) = [(0.297, 0.330, 0.363), (0.297, 0.330, 0.363), (0.675, 0.750, 0.825)]$, and maximum capacity is defined as $\tilde{b}_4 = (19,000; 20,000; 21,000)$. The *fifth constraint* corresponds to the capacity of public transportation, where

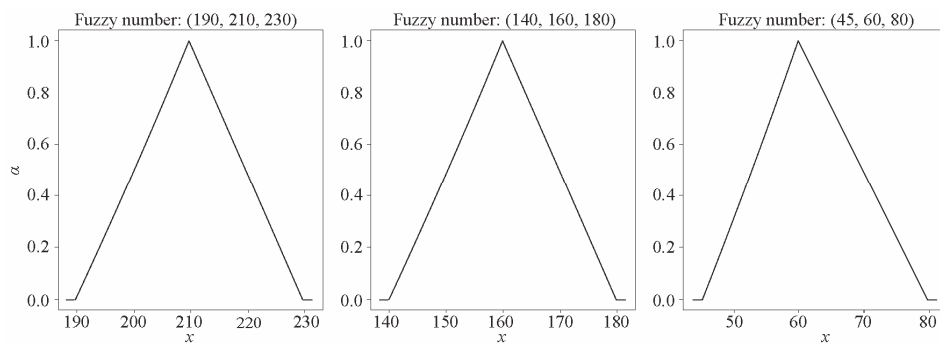
$(\tilde{a}_{51}, \tilde{a}_{52}, \tilde{a}_{53}) = \left[(0.9, 1.0, 1.1), (0.9, 1.0, 1.1), (0.9, 1.0, 1.1) \right]$, and maximum capacity is reflected by the fuzzy number $\tilde{b}_5 = (45,000; 46,000; 47,000)$. The *sixth constraint* is the daily quantity of waste production (in kg) imputable to tourists. The quantity for the three types of tourists is represented by the fuzzy numbers $(\tilde{a}_{61}, \tilde{a}_{62}, \tilde{a}_{63}) = \left[(2.1, 2.3, 2.5), (1.8, 2.0, 2.2), (0.9, 1.0, 1.1) \right]$, and maximum capacity is defined by the fuzzy number $\tilde{b}_6 = (90,000; 100,000; 110,000)$. The *last constraint* is associated with the maximum number of people who can enter St Mark's Square for security reasons. The entrance by type of tourist is specified by the fuzzy numbers $(\tilde{a}_{71}, \tilde{a}_{72}, \tilde{a}_{73}) = \left[(0.3, 0.4, 0.5), (0.2, 0.3, 0.4), (0.6, 0.7, 0.8) \right]$, and maximum capacity is indicated by the fuzzy number $\tilde{b}_7 = (90,000; 100,000; 110,000)$.

In this work, we simplified the original problem considering an FLP problem with fuzzy cost coefficients,

$$\begin{aligned} \max \tilde{\mathbf{z}} &\approx \tilde{\mathbf{c}}\mathbf{x} & (3) \\ \text{s.t. } \mathbf{Ax} &\leq \mathbf{b} \\ \mathbf{x} &\geq \mathbf{0} \end{aligned}$$

where $\mathbf{x} = (\text{TH}, \text{NTH}, \text{E})^T \in \mathbb{R}^3$ is the vector of decision variables. The cost vector $\tilde{\mathbf{c}}^T = (\tilde{c}_1, \tilde{c}_2, \tilde{c}_3)^T$ is defined using the fuzzy numbers $\tilde{c}_1 = (190, 210, 230)$, $\tilde{c}_2 = (140, 160, 180)$, and $\tilde{c}_3 = (45, 60, 80)$.

Figure 2. Graphical representation of the membership functions of the cost vector



The constraints are represented as follows:

$$\tilde{\mathbf{A}}\mathbf{x} = \begin{pmatrix} \text{TH} & \text{NT} & \text{E} \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0.75 & 0.65 & 0.2 \\ 0.33 & 0.33 & 0.75 \\ 1 & 1 & 1 \\ 2.3 & 2 & 1 \\ 0.4 & 0.3 & 0.7 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \quad \text{and} \quad \tilde{\mathbf{b}} = \begin{pmatrix} 14,400 \\ 20,400 \\ 24,000 \\ 20,000 \\ 46,000 \\ 100,000 \\ 100,000 \end{pmatrix}.$$

The FLP problem (3) can be transformed into its equivalent crisp LP problem replacing the fuzzy cost function by

$$\max \mathbf{z} = R(\tilde{c}_1)x_1 + R(\tilde{c}_2)x_2 + R(\tilde{c}_3)x_3$$

where R is one of the ranking functions defined in Table 2.

Table 3 presents the transformed cost functions using each ranking function from Table 2. The rightmost column displays the optimal values for each decision variable and estimated maximum benefit.

Table 3

Transformed cost functions

No.	Function's name	Transformed cost function	Solution: optimal values
1	Yager's F_1	$\max \mathbf{z} = 210x_1 + 160x_2 + 61.67x_3$	$x_1 = 14,400$ $x_2 = 16,252.4$ $x_3 = 13,179.6$ Maximum benefit = €6,437,130.9
2	Yager's F_3	$\max \mathbf{z} = 210x_1 + 160x_2 + 61.25x_3$	$x_1 = 14,400$ $x_2 = 16,252.4$ $x_3 = 13,179.6$ Maximum benefit = €6,431,639.4
3	Adamo ($\alpha = 0.5$)	$\max \mathbf{z} = 220x_1 + 170x_2 + 70x_3$	$x_1 = 14,400$ $x_2 = 16,252.4$ $x_3 = 13,179.6$ Maximum benefit = €6,853,485.2

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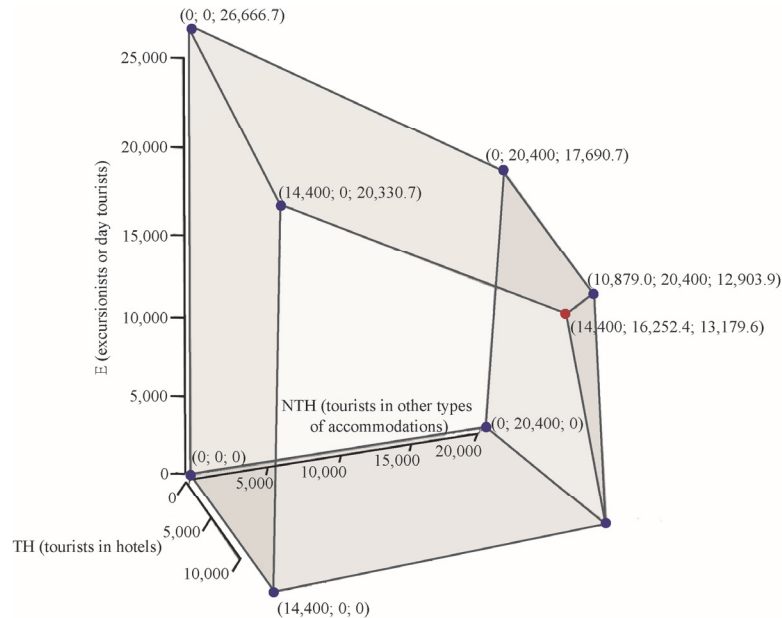
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No.	Function's name	Transformed cost function	Solution: optimal values
4	Campos ($\lambda = 0.5$)	$\max z = 216.67x_1 + 113.33x_2 + 45x_3$	$x_1 = 14,400$ $x_2 = 16,252.4$ $x_3 = 13,179.6$ Maximum benefit = €5,555,024.1
5	González ($\alpha = 0.5,$ $\lambda = 0.5$)	$\max z = 209.57x_1 + 160x_2 + 61.67x_3$	$x_1 = 14,400$ $x_2 = 16,252.4$ $x_3 = 13,179.6$ Maximum benefit = €6,430,870.0

Note. x_1 : optimal number of tourists in hotels; x_2 : optimal number of tourists in other types of accommodations; x_3 : optimal number of excursionists or day tourists.

Figure 3 gives a graphical representation of the feasible region and optimal values for the vector of decision variables $\mathbf{x} = (\text{TH}, \text{NTH}, \text{E})^T$. According to Table 3, in all cases, the optimal number of tourists in the Italian city of Venice (marked with red in Figure 3) is as follows: TH (tourists in hotels) = 14,400, NTH (tourists in other types of accommodations) = 16,252.4 (rounded to 16,252), and E (excursionists or day tourists) = 13,179.6 (rounded to 13,180).

Figure 3. Graphical representation of the feasible region and optimal values of vector \mathbf{x}



Given the uncertainty in the economic benefit generated by each tourist type (represented by the fuzzy cost vector $\tilde{\mathbf{c}}^T = (\tilde{c}_1, \tilde{c}_2, \tilde{c}_3)^T$), the estimation of the maximum benefit will depend on the selected ranking function. Considering the five ranking functions in Tables 2 and 3, the estimated maximum benefit generated by tourists in the Italian city of Venice varies from €5,555,024.1 to €6,853,485.2.

6. Conclusion

We focused on the solution for FLP problems in which the coefficients of the objective function (cost vector) are fuzzy numbers. The optimal solution was obtained using the ranking function approach, in which the original fuzzy problem is transformed into a crisp problem and then solved using standard methods.

We introduced a numerical example from the field of tourism where the goal was to maximize the economic benefit generated by tourists in the Italian city of Venice subject to different socio-environmental constraints. The benefit generated by each tourist type was assumed to be uncertain and represented by the fuzzy numbers $\tilde{c}_1 = (190, 210, 230)$, $\tilde{c}_2 = (140, 160, 180)$, and $\tilde{c}_3 = (45, 60, 80)$. Based on our results, the optimum number of visitors to Venice per day is 43,832 people, of whom 14,400 are tourists staying in hotels, 16,252 are tourists in other types of accommodations, and 13,180 are one-day tourists.

The estimation of the maximum benefit generated by tourists in Venice depends on the selected ranking function. Considering the five different ranking functions, this quantity varies from €5,555,024.1 to €6,853,485.2. Specifically, the estimated maximum benefit using Yager's F_1 function is €6,437,130.9; using Yager's F_3 is €6,431,639.4; using Adamo function ($\alpha = 0.5$) is €6,853,485.2; using Campos function ($\lambda = 0.5$) is €5,555,024.1; and using González function ($\alpha = 0.5, \lambda = 0.5$) is €6,430,870.

Finally, for practitioners, we recommend the use of different ranking functions for developing scenarios, such as a pessimist scenario (in our case, the minimum) and an optimist scenario (in our case, the maximum). From a computational point of view, we suggest the application of the FuzzyLP package implemented in R (Villacorta et al. [2017], R Core Team [2020]). However, for non-programmers we advise the use of Solver implemented in Excel.

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