Estimations of pooled dynamic panel data model with time-space dependence of selected Sub-Saharan African urban agglomerations, 2000–2020

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The ongoing progress in the spatial econometrics literature includes an eternal debate on suitable methods to comprehensively estimate time-space dependence. Nevertheless, techniques for carrying out such studies are inherently confined to purely cross-sectional or time-series studies. Thus, this paper reviewed different estimation techniques previously applied in the time-space literature, particularly the general dynamic panel model and approaches integrating pooled OLS, FE, RE, GMM-difference (Diff), and GMM-systematic (Sys) in estimating the time-space dynamic panel model. To achieve this objective, The authors determined the predictive efficiency of each method by evaluating the root mean square error (RMSE) estimators of Monte Carlo simulations of urban agglomeration spanning 2000–2020 for 22 Sub-Saharan African countries. The results indicate that the estimates arrived at using the conventional methods (pooled OLS, FE, and RE) are not consistent if endogeneity problems plausibly exist. Satisfactorily, the estimation procedures based on GMM methods, including the assumption of sequential exogeneity of the independent variables, offer ideal options for overcoming problems of endogeneity, heterogeneity, and feedback effects. Explicitly, the GMM-Sys estimators are characterized by low bias and excellent efficiency and are hence ideal for modeling spatial concepts with time-space dependence.
Introduction

Urbanization and the influx of populations from rural to urban regions have given rise to regional time-space dependence concepts such as urban agglomeration and regional economic performance, which are the major focus of global stakeholders and scholars (Castells–Quintana 2018, Moreno 2017). There is considerable concern regarding the unending debate about estimation procedures, measures of urban agglomeration, and urban socioeconomic needs such as access to water, electricity access, and sanitation, as well as whether there is a connection between the increase in urban populations and regional economic performance (Kanbur–Zhuang 2013). While urban regions act as a prospective basis for improving quality of life through greater employment incomes, there is an increasing economic disparity in terms of gross domestic product (GDP) per capita, especially in developing regions such as Sub-Saharan Africa (Manteaw 2020).

Urban agglomeration and regional economic performance indicators such as the urban share of the population, population density, and GDP per capita present a high level of measurement problems such as heterogeneity across cities, countries, and regions (Elhorst 2003, OECD 2011). Additionally, and most importantly, regions do not exist in isolation due to the mobility of factors such as labor following the migrating population from country to country, city to city, or region to region (EC 2014). Therefore, addressing the interaction effect between urban agglomeration and regional economic performance indicators and agents is one of the topics that spatial econometricians cover (Arbia–Prucha 2013).

It is, therefore, not surprising that a wide array of literature exists on urban agglomeration and regional economic performance, as spatial estimation logic is applied to concepts of time-space dependence, especially urban population and GDP per capita (Elhorst et al. 2007, Baltagi et al. 2012, Maket 2021). Nonetheless, and in glaring contrast, several studies have extended the estimation of concepts with time-space dependence by following a myriad of estimation procedures; hence, in this study, we extend this growing body of the literature by reviewing various procedures for estimating dynamic panel data (DPD) models and determining the predictive efficiency of each method used in estimating the dynamic panel model, as detailed later.

Empirical economic research entailing panel datasets with specified cross-sectional units and a given number of periods remains a prevalent research setting (Binder et al. 2005). Empirical findings from the standard DPD model indicate that integrated pooled ordinary least squares (OLS), fixed effects (FE), random effects (RE), and generalized method of moments (GMM) estimators are efficient and provide an ideal summary of dynamic panel models bearing both FE and RE (Kruiniger 2013, Hsiao–Pesaran 2004). More specifically, there is ongoing interest in estimating DPD models in the current period, with particular attention given to
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Spatial dependence concepts. These concepts arise from economic processes such as urban agglomeration, unemployment rate, urban development, per capita income, regional economic growth, and urban demographic changes, leading to the development of models referred to as spatial dynamic panel data (SDPD) models (Anselin et al. 2008, Lee–Yu 2012).

The growing attention on these approaches can be attributed to the fact that panel datasets provide scholars with lengthy prospect modeling instead of equation cross-sectional modeling, which has long been the primary focus in the literature on spatial econometrics. Notably, the DPD model in time and space is useful for addressing 1) the serial dependence between a spatial unit’s observations over a given period, 2) the spatial dependence between the observations at every point in time, 3) specific time period effects and unobservable spatial effects, and 4) the endogeneity of regressors apart from the explained variable lagged in time or space (LeSage–Pace 2009, Hsiao–Pesaran 2004, Arellano 2003, Baltagi 2005).

Baltagi (2005) describes the various benefits of employing DPD models in econometric analyses. The author argues that the DPD model uses informative panel data that are more variable, more efficient, and able to uncover the adjustment dynamics. This model allows the easy identification and measures of impact that are not measurable in a strictly time series or purely cross-sectional model. Along the same lines, Hsiao (2014) highlights that DPD models can control for individual heterogeneity and give rise to more degrees of freedom. Additionally, Hsiao (2014) suggests that DPD models reduce the biases arising from data aggregations over time and cross-sectional regions.

Despite the growing interest and the substantial benefits from time-space DPD models, various specification forms of the dynamic models have been proposed to address the resulting weaknesses and biases that accompany varying estimation structures and outcomes, leading to an unending progression of DPD model specification forms (Nickell 1981). This need to continuously revise the model can partly be attributed to the instances in which the dynamic process with time-space dependence occurs, since regions or countries do not have exactly the same structural features. Therefore, convergence tends toward different steady states compared to the per capita incomes and other key macroeconomic variables (Arbia et al. 2008).

Hsiao et al. (2002) proposed a transformed likelihood form of a DPD model based on the first difference. However, the drawback of this model form is the inability to determine the estimates of the time-variant explanatory variables. To correct this form of the dynamic model, Lee–Yu (2010) implemented a spatial dynamic panel model that permits time and spatial dependence and the component mixing of the space-time dependence. Debarsy et al. (2012) extended this technique by applying a space-time filter to the restraint on the mixing term that acts as a reflection. This kind of restraint speeds up the separation of space and time dependence that streamlines the estimation procedure, especially in the Bayesian Markov chain Monte Carlo
(MCMC) technique. In a more current form, Lee–Yu (2016) adopt a spatial Durbin DPD model, which simultaneously incorporates time dependence, space-time dependence, and spatial dependence on the regressors.

Building on this literature on the applicability of the model, this paper considers the time-space dependence specification of the DPD model, which is defined to consist of a temporal lag, which captures the time dependence, and a spatial lag, which captures the spatial dependence (Anselin 2001: 318.). A plethora of studies have theoretically applied this framework by building on a priori theory that does not include the \( WX \), which refers to a matrix defining the spatial framework of the independent variables in the model (Yu et al. 2008, Parent–LeSage 2012, Yu 2014, Yang 2018). In the case of cross-sectional data (when the model contains spatially lagged explained variables), Kelejian–Prucha (2002) recommend a 2SLS estimation procedure. They suggest that the technique set should be held to a small level to minimize any possible linear dependence; thus, they suggest that the independent variables and the matrix connecting the spatial units of the independent variables \( X, W_nX \) should be utilized whenever the number of independent variables is large. However, the inclusion of a spatial lag on the regressor impacts the performance of the estimation procedure (Pace et al. 2012).

Considering the works of Arellano–Bond (1991), Kapoor et al. (2007), and Baltagi et al. (2018), this paper seeks to integrate the panel OLS, RE, FE, and GMM methods in estimating the spatial panel data model with time-space dependence. These estimation models are selected to determine and compare the distinctive feature of the predictive efficiency of the FE, RE, and panel OLS models and are categorized as the conventional methods for estimating spatial panel data models. These models require strict exogeneity of the independent variables, which is easily violated. In contrast, the GMM-Diff and GMM-Sys is preferred because of its ability to overcome endogeneity problems and is regarded as suitable for dynamic models (Maulana–Aginta 2022, Jaber 2022, Purwono et al. 2021, Ramdhan 2021). In addition, we specify the overall time-space dependence dynamic model and its applicability form. Furthermore, we test its suitability using Monte Carlo simulation to determine the predictive efficiency of each method’s estimators by determining the root mean square error (RMSE) procedure in line with Kelejian et al. (2006). The following research hypotheses are tested in response to the study’s research objectives of specifying and estimating the spatial DPD model:

- **H₀**: There is no significant difference in the predictive efficiency of the dynamic panel estimation procedures (conventional and dynamic methods).
- **H₁**: There is no significant difference in the bias of the coefficient estimates of the estimation methods (conventional and dynamic methods).

The paper proceeds as follows: The introduction provides a general overview of the DPD models, their applied forms, and the dimensions examined by previous researchers. First, we provide a generalized dynamic panel time-space dependence
data model and its specification form. Then, we justify integrating the pooled OLS, FE, RE, and GMM estimation methods. Subsequently, we present a detailed strategy for estimating the dynamic panel time-space dependence data model and its overall applicability. Later, we provide the simulation design and build up of the dynamic panel time-space dependence data models for simulation and further estimation by following the Monte Carlo simulation technique. Additionally, this section describes data simulation processing and sampling. We present the Monte Carlo simulation results of the Stata-generated $S = 1000$. Finally, we draw deductions based on the reviewed literature, model specification forms, and simulation results as applied to the urban agglomeration dynamics of 22 Sub-Saharan African countries (see Figure A1 in the Appendix).

**Generalized dynamic panel time-space dependence model**

This section presents the foundation of the generalized dynamic panel model in time and space (time-space dependence), which generalizes the foundational model that has been considered in the econometric and spatial literature concepts. We argue that the generalized dynamic panel model suffers from identification pitfalls. Thus, it may not be suitable for empirical analysis in its simpler specification and in isolation from the various estimation forms and conditions embedded in it. Nevertheless, when the estimation models are arranged well in their specification forms, estimation framework, and amplified nexus, this process helps single out a suitable estimation model and its ideal specification form for studying the time-space data in line with the aim of the empirical study.

In accordance with the postulation of Elhorst (2014), the generalized dynamic panel model in time and space presented in vector form for cross-sections ($i$) of the observations at a particular time ($t$) is simplistically presented as follows:

$$Y_t = \tau Y_{t-1} + \delta Y_t + \eta W Y_{t-1} + X_t \beta_1 + W X_t \beta_2 + X_{t-1} \beta_3 + W X_{t-1} \beta_4 + Z_t \theta + \nu_t$$  \hspace{1cm} (1A)  

$$\nu_t = \gamma \nu_{t-1} + \rho W \nu_t + \mu + \lambda_{it} + \varepsilon_t$$  \hspace{1cm} (1B)  

$$\mu = kW \mu + \rho W \nu_t + \xi$$  \hspace{1cm} (1C)  

where $Y_t$ is the denotation of an $(N \times 1)$ vector that comprises a single observation of the explained variable for each spatial unit, $(i = 1,2,3,\ldots,N)$ in the sample at time $t$, $(t = 1,2,3,\ldots,T)$, $X_t$ is the denotation of an $(N \times K)$ matrix of the regressor, and $Z_t$ is the denotation of an $(N \times L)$ matrix of the dependent variables. Furthermore, the $(t-1)$ subscript denotes a vector matrix of serially lagged value, whereas a matrix multiplied by $(W)$ represents the spatially lagged value. The $(N \times N)$ matrix $(W)$ is a positive matrix that denotes a constant that explains the spatial framework of the sample units. Its diagonal components are assumed to be 0 (zero) because no spatial unit can be its neighbor. The response parameter estimates of the successive explained variable lagged in time $(Y_{t-1})$, explained variable lagged in space $(WY_t)$, and
explained variable lagged in both time and space \((WY_{t-1})\) are denoted by \((\tau, \delta, \text{ and } \eta)\) scalers. \((K \times 1)\) is the vector for \((\beta_1, \beta_2, \beta_3, \text{ and } \beta_4)\) containing the response parameter estimates of the independent variables and the \((L \times 1)\) vector \((\theta)\) of the dependent variables in the general estimation model \((1A)\).

The \((N \times 1)\) vector \((\nu)\) represents the random error term presumed to be both serially and spatially correlated. \((\rho)\) represents the serial autocorrelation coefficient estimate, and \((\rho)\) represents the spatial autocorrelation coefficient estimate. The \((N \times 1)\) vector for \(\mu = (\mu_1, \ldots, \mu_N)^T\) contains the specific spatial effects denoted as \((\mu)\) and is purposed to control for every spatially specific, time- nonvarying variable whose omission could result in spurious findings in a classic cross-sectional study (Baltagi 2005). In the same supposition, \((\lambda)\) \((t = 1, \ldots, T)\) is the notation of time period-specific effects, where \((\mathcal{N})\) is the vector aimed at controlling for every time-specific, unit-nonvarying variable whose omission could lead to biased estimates in a classic time-series study. These spatial-specific effects and time period-specific effects may be treated as random or fixed effects. Additionally, the spatial-specific effects are spatially autocorrelated with the spatial autocorrelation coefficient \((k)\). Finally, \(\varepsilon_t = (\varepsilon_1 t, \ldots, \varepsilon_N t)^T\) and \((\xi)\) are the vectors representing i.i.d. stochastic terms, whose elements have 0 mean and finite variance as \((\delta^2)\) and \((\delta^2)\), respectively. The subsequent sections present a more logical understanding of how to state, specify, and apply the DPD model in estimating time-space dependence concepts such as urban agglomerations and regional economic performance.

**Specification of the dynamic panel time-space dependence data model**

In this section, our exposition and model specification are based on various assumptions and conditions that must be fulfilled. First, we assume that the differences in the exogenous variable will continue as the outcome of equilibrium to fundamental and nonvarying causes. Thus, we presume that the \((N \times 1)\) vector of the explained variable, \((Y)\), where \((N)\) is the representation of the number of regions, at a time period \((t)\) will continue at a dynamic and steady level so that \((Y_t = Y_{t-1})\), except when there is a change in the dynamics that influence the dependent variable \((Y)\). For instance, there could be variations in the independent variables \((X)\), whereby \(X_t = (x_{1t}, \ldots, x_{Nt})^T\) is the \((N \times K)\) matrix of the independent variable or in any other variations, such as unobservable effects. If such changes occur at a time \((t)\) and are transient, then \((Y_t \neq Y_{t-1})\), but provided a subsequent time period of latency as \(t \rightarrow T\), we once more anticipate that \((Y)\) will converge to a new equilibrium at which \((Y_T = Y_{T-1})\).
By assuming that the datasets are observed whereby \( Y_t \neq Y_{t-1} \) but tend to congregate, such as \( Y_t = f(Y_{t-1}) \), an autoregressive process is presumed:

\[
Y_t = \zeta + \gamma Y_{t-1}
\]  

(2)

where \( \zeta \) is an \( (N \times 1) \) vector and the scalar parameter is represented by \( \gamma \). With \(|\gamma| < 1\) and with no sequential variations, the dynamic autoregressive process tends to converge to \( Y_t = \frac{\zeta}{1 - \gamma} \) in the long run. Next, we consider the spatial connection between areas in the form of the matrix \( (W_N^s) \), which represents the time-nonvarying \( (N \times N) \) matrix. To explain the parameter estimates, we normalize \( (W_N^s) \) by dividing \( (W_N^s) \) by the maximum eigenvalue of \( (W_N^s) \) to give \( \frac{(W_N^s)}{\lambda_{\max}} \) or by dividing every element of \( (W_N^s) \) by its sum. These normalization strategies give rise to a maximum eigenvalue of \( \frac{\lambda_{\max}}{\lambda_{\min}} = 1 \) and the incessant range for which \( (I_N - \rho_1W_N) \) is considered nonsingular, that is, \( \frac{1}{\min(eig)} < \rho_1 < \frac{1}{\max(eig)} = 1 \), whereby \( \rho_1 \) is the scalar representing the spatial autoregressive parameter estimate. Thus, given Equation (2) above, logically, we have

\[
\rho_1W_NY_t = \rho_1W_N\zeta + \rho_1W_NY_{t-1}
\]  

(3)

By subtracting (3) from (2), we develop another logically aligned expression whereby the time-space dependence signified by (3) can be traced in (4) and (5) as an explicit contributing factor to the variations in \( (Y_t) \). Therefore, we have

\[
Y_t - \rho_1W_NY_t = \zeta + \gamma Y_{t-1} - (\rho_1W_N\zeta + \rho_1W_NY_{t-1}).
\]

This can be simplified further by combining like terms and factoring out the common factor. The equation then becomes

\[
(I_N - \rho_1W_N)Y_t = (\gamma I_N - \rho_1W_N)Y_{t-1} + (I_N - \rho_1W_N)\zeta
\]  

(4)

Furthermore, by taking \( \theta = \rho_1\gamma \), we obtain

\[
Y_t = B_N^{-1}[C_NY_{t-1} + B_N\zeta]
\]  

(5)

where \( B_N = (I_N - \rho_1W_N), \ C_N = (\gamma I_N + \theta W_N) \), in which \( \gamma \) is the parameter of the autoregressive time dependence, \( \rho_1 \) is the coefficient of spatial lag, \( \theta \) is the parameter of time-space diffusion and \( (I_N) \) is the \( N \times N \) order identity matrix. After solving Equation (4) above and given an ideal restriction of the parameters, Equation (4) converges to

\[
Y_t = (B_N - C_N)^{-1}B_N\zeta.
\]

Introducing the extra covariates by rewriting \( B_N\zeta = (X\beta) \) in Equation (4), whereby \( \beta \) is the \( (k \times 1) \) vector, results in

\[
Y_t = B_N^{-1}[C_NY_{t-1} + X\beta].
\]

(6)

To maintain a stable and dynamic estimation following Elhorst (2014), Parent–LeSage (2012), and Debarsy et al. (2012), a large characteristic root \( (\text{eig}_{max}) \) of \( (B_N^{-1}C_N) \) is required to be less than 1. This restraint guarantees that \( (Y_t) \) congregates to the stability level where \( Y_t = (B_N - C_N)^{-1}X\beta \). To further add to the realism of this model, the following steps are taken. First, the restriction requirement that \( (\theta = -\rho_1\gamma) \) is removed given that \( \rho_1 \) and \( \gamma \) are not known so that \( \theta \) can freely vary. Nonetheless,
we expect that \( \tilde{\theta} \approx -\tilde{\beta} \hat{y} \). Second, the time-nonvarying matrix \( (X) \) is replaced with a time-varying matrix \( (X_t) \). Third, the error term \( (\varepsilon_t) \) represents the unobservables.

Although the system may, depending on \( (\varepsilon_t) \), still tend toward the equilibrium level, the stability will be disturbed unceasingly until a new stability level is reached as \( (\varepsilon_t) \) fluctuates. Simplistically and for model estimations, spatial interconnectivity is presumed to be static throughout. These specifications result in the time-space dynamic panel model for region \( (i = 1, 2, 3, \ldots, N) \) and period \( (t = 1, 2, 3, \ldots, T) \), which is stated as

\[
Y_{it} = \gamma Y_{i,t-1} + \rho_1 W_i Y_t + \theta W_i Y_{i,t-1} + X_{it} \beta + \varepsilon_t; \quad \varepsilon_{it} = \alpha_i + \mu_{it} \tag{7}
\]

where \( (\varepsilon_{it}) \) is the stochastic term for region \( (i) \) and period \( (t) \) and \( (W_i = W_{i1}, \ldots, W_{iN}) \) is the \((1 \times N)\) vector corresponding to the \( i \)-th row of matrix \( W_N \). By grouping the spatial panel data by each time \( (t) \), we obtain

\[
Y_t = \gamma Y_{t-1} + \rho_1 W_N Y_t + \theta W_N Y_{t-1} + X_t \beta + \varepsilon_t \tag{8}
\]

and this can be simplified as

\[
[-C_N L + B_N] Y_t = X_t \beta + \varepsilon_t \tag{9}
\]

where \( (\varepsilon_t = \varepsilon_{it1}, \ldots, \varepsilon_{itN})^T \) is an \((N \times 1)\) vector and \((L)\) is the time-lag operator, that is, \( L Y_t = Y_{t-1} \). \((B_N)\) is the nonsingular matrix, and \((B_N^{-1})\) is bounded uniformly.

In strict logic, \( (\alpha_t) \) in Equation (7) is referred to as the fixed effect if it can freely correlate with all of the independent variables \( (X_{it}) \), and it becomes a random effect if it is distributed independently. Additionally, \( (\alpha_t) \) is correlated with lagged endogenous variables by design. As mentioned above, this paper attempts to look at the dynamic panel fixed and random effect models whereby a subset, but not all, of the independent variables are correlated with \( (\alpha_t) \). Throughout this paper, we anchor our assertions on the assumptions below:

**Assumption 1**: The disturbance term \( (\mu_t) \) and unobserved unit-specific effects \( (\alpha_t) \) are distributed independently across regions \( (i) \) and thus meet the conditions that \( E[\mu_{it}] = 0, E[\mu_{is} \mu_{it}] = 0 \qquad \forall s \neq t, \) and \( E[\alpha_t \mu_{it}] = 0 \). This condition depicts that the parameter estimates \( (\gamma, \theta, \text{and } \beta) \) hinge on the assumption regarding the relatedness between the independent variables and unit-specific effects.

**Assumption 2**: The independent variable is decomposable as \( X_{it} = (X_{it1}, X_{it2})' \), such that \( E[\alpha_t | X_{it1}] = 0, E[\alpha_t | X_{it2}] \neq 0 \) and \( E[\alpha_t] \neq 0 \).

**Assumption 3**: The time-varying independent variables \( (X_{it}) \) are strictly exogenous such that \( E[\mu_{it} | Z_{it0}, Z_{it1}, ..., Z_{iT}] = 0 \) or straightforwardly predetermined as \( E[\mu_{it} | X_{it0}, X_{it1}, ..., X_{it6}; \alpha_i] = 0 \) and \( E[\mu_{it} | X_{it}] \neq 0 \quad \forall s > t \). However, Assumption 3 is largely violated due to the endogeneity problem in time-space variables, as considered in this study’s case.
Dynamic panel data model estimation framework

Rewriting the time-space dynamic panel model specified in Equation (7) in a longitudinal framework, we simplistically present the empirical time-space dynamic panel model of focus as follows:

\[ Y_{it} = \alpha + \beta X_{it} + \epsilon_{it}, i = 1, ..., N, t = 1, ..., T \]  \hspace{1cm} (10)

In this case, \((N)\) represents the number of regions under consideration over time \((T)\), the subscripts \(i\) and \(t\) represent the \(i^{th}\) region, and \(t^{th}\) represents the time. In this section, we present the estimation framework of the time-space DPD model with a special focus on controlling for endogeneity and considering short panels, where \((N)\) is much larger than \((T)\), as is the case in spatial econometric studies. Therefore, as mentioned above, all the estimations and results under the subsequent sections are anchored on the assumptions that \((T)\) is fixed and \((N) \to \infty\) or, more informally, \((T)\) is fixed and \((N)\) is adequately large.

Breaking down the error term \((\epsilon_{it})\) in Equation (10) into \(\epsilon_{it} = \eta_{i} + \mu_{it}\) represents the unobserved heterogeneity resulting from explicit modeling of variables that are not observable either because of insufficient data or because the variables are inherently not observable. This breakdown results in the following extended model specification form:

\[ Y_{it} = \alpha + \beta X_{it} + \eta_{i} + \mu_{it}; i = 1, ..., N, t = 1, ..., T \]  \hspace{1cm} (11)

where \((\eta_{i})\) is the representation of the unobserved heterogeneity of the regions in the estimation sample and \((\mu_{i})\) is the model’s stochastic term. In this study, \((\eta_{i})\) is assumed to vary freely between regions and not over the sample period. Practically, this means that this variable captures the unobserved heterogeneity linked to region \((i)\) that is nonvarying over the sampling period. Furthermore, the inclusion of \((\eta_{i})\) introduces the traditional control variables that need to be included in the estimation models, which are generally grouped into FE and RE. In both FE and RE, the persistent estimation of coefficient estimate \((\beta)\) depends fundamentally on the noncorrelation assumption between the error term \((\mu_{i})\) and the explanatory variable \((X_{i})\). Nonetheless, RE uses the conventional assumption of noncorrelation between \(X_{i1}, ..., X_{iT}\) and \((\eta_{i})\) specific effects. This will be detailed later, but before doing so by specifying the estimation framework of the dynamic panel model under different methods, it is paramount to understand the underlying assumptions of stationarity and how it is dealt with.

Apart from the listed assumptions 1–3, throughout this study, we consider the assumptions of random sample space and stationarity and the unit root of the model estimators. With regard to the random sample, we entirely assume that the sample with a finite fourth-order moment is defined as \(\{\eta_{0}, Y_{0}, Y_{1}, ..., Y_{T}\}_{t=1}^{N}\). Let us call this core assumption \(A\) which satisfies the conditional assumption that \(E[\mu_{it} | \eta_{0}, Y_{0}, Y_{1}, ..., Y_{(t-1)}] = 0\), and \((t = 1, ..., T)\). This is the main assumption...
throughout this study, as we consider the panel estimators that are consistently reliable and asymptotically normal for adequately large \( N \) and fixed \( T \). Note also that we treat the error term to have unconditional variance expressed as \( E(\eta^2) = \sigma^2 \) and to have the freedom to vary with time \( T \) and to be different from the conditional variances, expressed as

\[
E(\eta^2 | \eta, Y_t^0, Y_{t+1}, \ldots, Y_{t(T-1)})
\]  

(12)

Regarding the stability and stationarity in the mean condition, let the matrix of \((\eta, Y_t^0)\) be represented as follows: \( \text{Var} \left( \begin{array}{c} \eta \\ Y_t^0 \end{array} \right) = \left( \begin{array}{c} \sigma^2_y \\ Y_0 Y_0' \end{array} \right) \). In this case, \((\gamma_0)\) and, \((\gamma_0)\) are treated as parameter estimates (Alvarez–Arellano 2022). More specifically, under this assumption, we consider the possibility of identification failures when the dynamic panel estimation autoregressive process contains an unit root and considering the stationarity in mean traits. To execute this assumption, we consider a case where dynamic estimators allow for panel heteroscedasticity but still exploit the condition of stationarity in the mean such that for each \((t)\), the mean of the observed value of the dependent variable \( Y_{it} \) that is conditioned on \( (\eta, Y_{i0}, Y_{i1}, \ldots, Y_{i(T-1)}) \) coincides with the mean of the steady state in the process of \( \mu_i = \left( \frac{\eta}{1 - a_{\ell \rho}} \right) \). Thus, we assume that

\[
\gamma_0 = \begin{pmatrix} \text{Cov}(\eta, Y_{1(1-\rho)}) \\ \vdots \\ \text{Cov}(\eta, Y_{0}) \end{pmatrix} = \sigma^2_y \left( 1 - a_{\ell \rho} \right)^L \rho.
\]

Let us call this core assumption B. Therefore, under maintained assumptions A and B above, the relationship between the observed values of the dependent variable \( Y_{it} \) and the FE \( (\eta) \) does not rely on time. Thus, the first differenced data become orthogonal to the FE represented by \( (\eta) \). This leads to the orthogonality condition for the error levels utilized in the GMM-Sys method (Arellano–Bover 1995, Blundell–Bond 1998). This conditional assumption is largely simplified under the estimation framework of the dynamic panel model estimation methods, as outlined in the subsequent paragraphs (Klutse et al. 2022, Kuncoro 2020).

As mentioned above, the dynamic panel model under consideration assumes both fixed and random effects, and to estimate the coefficient estimates of the model represented by Equation (11) using the FE and RE frameworks, the assumption of strict exogeneity and feedback effects should be held so that

\[
E(\mu_{it}) | X_{i1}, X_{i2}, \ldots, X_{iT}, (\eta_0) = 0.
\]  

(13)

This supposition neglects any possible correlation between current residuals and present or future values of the independent variables (the feedback effect phenomenon, which moves from dependent variable \( Y \) to regressor \( X \)); if this
phenomenon is present, the FE and RE estimates become spurious or inconsistent (Nickell 1981).

The estimation of the coefficient estimates of Equation (11) using the panel OLS, is generally known as pooled OLS, which ignores the heterogeneity problem; the following sufficient requirement should be met:

$$E(\varepsilon_{it} | X_{it}) = 0.$$  \hspace{1cm} (14)

However, this supposition is not as restrictive as that of strict exogeneity. It thus can easily be violated if there is an unobserved effect ($\eta_i$) correlated with the independent variables contained in the error term ($\varepsilon_i$). Additionally, pooled OLS produces inconsistent estimates because the lagged endogenous variable $Y_{it-1}$ is correlated with the unobserved fixed effect ($\eta_i$), that is, $E[Y_{it-1}\eta_i] \neq 0$, thereby overestimating the coefficient estimates (Baltagi–Liu 2008). Therefore, a natural mitigation to the problems of exogeneity and heterogeneity (as required to be met in RE, FE, and pooled OLS estimations) is the use of the instrumental variables outside the estimation model in question. This approach leads us to the GMM estimation model, which is ideal for short panel datasets and uses sequential exogenous variables as the instruments. These models are classified into two categories based on GMM: estimators and instrumental variables. In this study, these methods are designed to estimate time-space dynamic panel models, including one or two lagged response variable values as part of the exogenous variables (Bond 2002, Dang et al. 2015).

The estimation of the time-space DPD model following the GMM framework is anchored in the development of Arellano–Bond (1991), referred to as the Arellano–Bond estimator or first-difference GMM (GMM-Diff). This estimation procedure computes the difference between every variable and its first lag. Thus, by rewriting Equation (11) and by introducing the lagged value of the dependent variable ($Y_{it-1}$) on the right-hand side (RHS), we have

$$\Delta Y_{it} = \alpha_0 + \alpha_1 \Delta Y_{it-1} + \beta \Delta X_{it} + \Delta \varepsilon_{it}, \hspace{1cm} i = 1, \ldots, N, t = 1, \ldots, T$$  \hspace{1cm} (15)

This estimation procedure considers the moment condition, $E[Y_{it-S} \Delta \varepsilon_{it}] = 0$, with $t = 3, \ldots, T$ and $S = 2, \ldots, t-1$, and utilizes the vector $(Y_1, \ldots, Y_{t-2})$ as the GMM instruments for $\Delta Y_{it-1}$ in Equation (11). Moreover, Blundell–Bond (1998) developed the system GMM estimator (GMM-Sys), which is critical for enhancing the efficiency of the GMM-Diff by using additional moment conditions in the level Equation (8). The procedure regards $(\Delta Y_{t-2}, \ldots, \Delta Y_{t-1})$ as the instruments for ($Y_{it-1}$) under the moment condition $E[Y_{it-2} \Delta \varepsilon_{it}] = 0$, with $t = 3, \ldots, T$ and $s = 1, \ldots, t-2$ (Arellano–Bover 1995, Blundell–Bond 1998). Furthermore, the GMM-Sys can exploit the moment condition:

$$E[\Delta X_{it-1} \eta_i + \mu_{it}] = 0$$  \hspace{1cm} (16)

These GMM methods address the unit root or stationarity trait discussed in the previous section and thus ensure correctly estimated coefficient estimates of the dynamic panel model.
Explicitly, we can model the time-space dynamic panel model by extending the model in Equation (12) and decomposing the error term ($\varepsilon_{it}$) into three components: the unobserved heterogeneity ($\eta_i$) of the regions ($i$); ($\lambda_t$), which represents the time fixed effects; and ($\mu_{it}$), representing the model’s error term. Thus, the overall econometric estimation time-space DPD model is presented as follows:

$$ Y_{it} = \alpha_0 + \alpha_1 Y_{it-1} + \beta X_{it} + \eta_i + \lambda_t + \mu_{it}, i = 1, ..., N, t = 1, ..., T $$  \hspace{1cm} (17)

If the model represented in Equation (17) is ideal, with $\alpha_1 \neq 0$, the omission of $(Y_{i,t-1})$ in the regression will render the coefficient estimator ($\beta$) spurious if $(Y_{i,t-1})$ (which is included as part of the error term of the model) correlates with the exogenous variable ($X_{it}$). Of great importance is that the estimation of the model in Equation (17) cannot be adequately estimated by a framework that requires the strict exogeneity trait of regressors, as is the case for RE and FE estimation methods, because $(Y_{i,t-1})$ is not a strictly exogenous variable. However, if the independent variables are sequentially exogenous, the model estimates in Equation (17) can be estimated by GMM frameworks, as discussed above.

**Monte Carlo simulation**

Monte Carlo simulation is conducted to analyze the suitability of the proposed specified DPD model. Thus, in this section, we provide the framework for creating the simulated panel sample data with similar attributes to those available for regional spatial studies. In this example, we apply the Monte Carlo process and steps to generate a set of random samples based on the model specified that blends the features mentioned above in the most elaborate manner. Specifically, the Monte Carlo simulation strategy focuses on uncovering the efficiency of the estimator performance of pooled OLS, FE, RE, GMM-Diff, and GMM-Sys techniques by determining the RMSE. Additionally, the simulation focuses on the specific case of determining the problems related to the endogeneity, heterogeneity, and temporal persistence of explained variables and feedback effects.

The general model for carrying out the simulation is an extension of the model in Equation (17) but with the exclusion of the intercept ($\alpha_0$) for the purposes of simplicity and without losing preponderance, where ($\mu_{it}$), is the random error term. The simulation model is presented as follows:

$$ Y_{it} = \alpha Y_{it-1} + \beta X_{it} + \eta_i + \lambda_t + \mu_{it}, i = 1, ..., N, t = 1, ..., T $$  \hspace{1cm} (18)

The model captures the dynamic behavior of the endogenous variable represented as $(Y_{i,t-1})$, the region’s heterogeneity represented as $(\mu_{it})$, the impact of the unobserved FE represented as $(\lambda_t)$, and the error term represented as $(\eta_i)$. As important as modeling the dynamic behavior of the dependent variable is, so is the modeling behavior of the regressor ($X_{it}$). Thus, the general time-space dynamic panel model for the regressor is presented as follows:
Estimations of pooled dynamic panel data model with time-space dependence of selected Sub-Saharan African urban agglomeration, 2000–2020

where \((\eta_i)\) captures the unobservable time-nonvarying region’s characteristics and exists if \(\tau \neq 0\). The unobservable time effect is represented by \((\Phi_\lambda t)\) relative to the value of \((\Phi)\). The dynamic endogeneity problem linked to the feedback effect from variable Y to X is represented by \((\theta_2\mu_{it-1})\), and its level of magnitude depends on the value of \((\theta)\).

The models in Equations (18) and (19) enable us to comprehensively determine significant estimator performance and the highlighted problems of endogeneity and heterogeneity. To achieve these objectives, it is paramount to rearrange the models into one general estimation model represented by Equation (20) and the parameters selected for illustration purposes. The model is presented as follows:

\[
Y_{it} = aY_{it-1} + \beta X_{it} + \eta_i + \lambda_t + \mu_{it} \\
X_{it} = \rho X_{it-1} + \eta_i + \Phi \lambda_t + \theta_1 \mu_{it} + \theta_2 \mu_{it-1} + \epsilon_{it}
\]

where \(\eta_i \sim N(0, \sigma_\eta^2)\), \(\lambda_t \sim N(0, \sigma_\lambda^2)\), \(\mu_{it} \sim N(0, \sigma_\mu^2)\), and \(\epsilon_{it} \sim N(0, \sigma_\epsilon^2)\) (assumed to follow a normal distribution with 0 mean and \(\infty\) variance). Furthermore, we assume that \(\sigma_\eta^2 = \sigma_\epsilon^2 = \sigma_\lambda^2 = \sigma_\mu^2 = 1, \beta = 1, \tau = 0.7, a = 0.5, \theta_1 = 0.6, \theta_2 = 0.5, \rho = 0.5\) and \(\Phi = 0.5\).

As shown in Model (18), we also assume that \(\eta_i, \lambda_t, \mu_{it}\) and \(\epsilon_{it}\) are normally distributed random variables.

To determine the efficiency of the estimation model method and estimation itself (pooled OLS, FE, RE, GMM-Diff, and GMM-Sys), the distance between the actual value and the estimated value is determined by calculating and evaluating the magnitude of the RMSE linked to the estimator of the \((\beta)\) coefficient estimate. The RMSE of the coefficient estimates of \((\beta)\) is calculated as follows:

\[
RMSE = \frac{\sum_{j=1}^{S} (\hat{\beta}_j - \beta)^2}{S},
\]

where \((\hat{\beta}_j)\) is the parameter estimate in the \((j-th)\) total simulated sample \((S)\). In our case, \((\beta = 1)\) and \(S = 1000\). Note also that this condition is applicable in calculating the RMSE for \((a)\).

**Empirical application: Dynamics of urban agglomeration in Sub-Saharan-Africa**

This section presents the suitability and predictive efficiency of the specified DPD model with time-space dependence following the estimation and Monte Carlo simulation frameworks. To effectively establish the efficiency of the DPD models, panel data from 22 Sub-Saharan African countries spanning from 2000 to 2020 are collected from the World Bank’s World Development Indicators (WDIs) through its official website for the period spanning from 2000 to 2020. The study utilizes two key variables: *regional economic performance* measured by income per capita as the dependent variable supposedly predicted by *urban agglomeration* (concentration of people in urban regions) (Lengyel–Szakálné 2012, Maket et al. 2022).
The data were collected and measured in absolute terms, which were log-transformed to reduce the likelihood of bias and endogeneity traits before estimation. The second phase of data processing was generating or simulating the *income per capita* by carrying out 1000 replications (Kocziszky et al. 2018). To this end, using the simulation function (*$= NORM.S.INV(RAND())$*) in Excel and Stata, a randomly sampled data series was generated within the sampling period of 2000–2020 for the 22 countries. Subsequently, the generated random series was added to the estimated series of the regional economic performance (*income per capita*) ($\bar{Y}$) to obtain the simulated panel series of *income per capita* ($Y$). The simulated panel series was replicated to obtain a simulated sample size of ($S = 1000$) for precision purposes and then stored in Excel for further estimation in Stata software.

In this application, the study considered the analysis of the Monte Carlo simulated panel data series using an integration of pooled OLS, RE, FE, GMM-Diff, and GMM-Sys estimation models, as discussed in the previous sections. Specifically, this study involved a general analysis of the simulated data and specific cases of heterogeneity and endogeneity assumptions following the previously defined presumptive moment conditions. The analysis and estimations were conducted using Excel, Stata, and Eviews software for comparison and evaluation purposes and to ensure the generalizability of the estimates.

**General estimates' performance of the simulated panel data**

In regard to the general analysis of the simulated panel data series, the overall performance of the coefficient estimates was evaluated following the model in Equation (20), which was plagued by diverse problems of endogeneity. To estimate the performance using pooled OLS, the omitted variable bias represented by ($\lambda$) is addressed by including the dummy variable and forcibly overlooking other problems. The other estimation models consider all the assumptions discussed previously. The estimation results of pooled OLS, RE, FE, GMM-Diff, and GMM-Sys are presented in Table 1. As indicated by the results, the GMM-Diff estimator appropriately addresses the endogeneity sources included in the general estimation Model (20) by eliminating the unobserved heterogeneity using the lagged values of explained and explanatory variables. The coefficient estimate $\hat{\beta} = 0.7163$ is much closer to the true estimate $\beta = 0.7789$ than are the pooled OLS, RE, and FE model estimates. Nonetheless, the estimate of intercept $\alpha = -4.4623$ is less satisfactory, although it is close to the true value of $\alpha = -6.3092$ on average since the value varies from $-11.4277$ to $2.5031$. More satisfactory are the GMM-Sys estimators, which have almost null biases. Specifically, the coefficient estimate is $\hat{\beta} = 0.7716 \approx 0.7789$, and $\alpha = -5.3486 \approx -6.3092$. This finding can be attributed to the fact that the GMM-Sys uses additional instruments on the basis of sequential endogeneity of the independent variables.
Simulated results for general model coefficient estimate performance

<table>
<thead>
<tr>
<th>Model</th>
<th>Parameter</th>
<th>Mean</th>
<th>SD</th>
<th>Minimum</th>
<th>Maximum</th>
<th>RMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pooled OLS</td>
<td>(\alpha)</td>
<td>-6.8180</td>
<td>15.6526</td>
<td>-8.3519</td>
<td>-5.2840</td>
<td>0.9783</td>
</tr>
<tr>
<td></td>
<td>(\beta)</td>
<td>0.8198</td>
<td>1.0409</td>
<td>0.7178</td>
<td>0.9218</td>
<td>0.9783</td>
</tr>
<tr>
<td>RE</td>
<td>(\alpha)</td>
<td>-6.8180</td>
<td>13.1599</td>
<td>-8.1076</td>
<td>-5.5283</td>
<td>0.9694</td>
</tr>
<tr>
<td></td>
<td>(\beta)</td>
<td>0.8198</td>
<td>0.8636</td>
<td>0.7352</td>
<td>0.9045</td>
<td>0.9694</td>
</tr>
<tr>
<td>FE</td>
<td>(\alpha)</td>
<td>-3.3259</td>
<td>82.5074</td>
<td>-11.4115</td>
<td>4.7600</td>
<td>0.9763</td>
</tr>
<tr>
<td></td>
<td>(\beta)</td>
<td>0.5870</td>
<td>5.4870</td>
<td>0.0493</td>
<td>1.1247</td>
<td>0.9763</td>
</tr>
<tr>
<td>GMM-Diff</td>
<td>(\alpha)</td>
<td>-4.4623</td>
<td>71.0766</td>
<td>-11.4277</td>
<td>2.5031</td>
<td>1.3241</td>
</tr>
<tr>
<td></td>
<td>(\beta)</td>
<td>0.7163</td>
<td>4.7441</td>
<td>0.2514</td>
<td>1.1812</td>
<td>1.3241</td>
</tr>
<tr>
<td>GMM-Sys</td>
<td>(\alpha)</td>
<td>-5.3486</td>
<td>54.2426</td>
<td>-10.6643</td>
<td>-0.0329</td>
<td>0.9704</td>
</tr>
<tr>
<td></td>
<td>(\beta)</td>
<td>0.7716</td>
<td>3.6264</td>
<td>0.4162</td>
<td>1.1270</td>
<td>0.9704</td>
</tr>
</tbody>
</table>

Note: SD, standard deviation; min, minimum; max, maximum; RMSE, root mean square error.
Source: Author’s estimations (2022).

Correlation between the explanatory variable and the unobserved heterogeneity

In this case, we examine the behavior of a reduced dynamic panel model, isolating the probable problems of endogeneity exhibited in the real data. Realistically, we focus on the unobserved heterogeneity problem associated only with the regressor \(X\), removing any other significant source of endogeneity. Under this scenario, we develop a newly modified simulation analysis that has changed the parameters in the general model, allocating some parameters to zero. For instance, in this case, the intercept parameter \(\alpha = 0\); thus, the regressor in this case becomes \(X\) only. Therefore, the estimation under this case follows the model captured as follows:

\[
\begin{align*}
Y_{it} &= \beta X_{it} + \eta_t + \mu_t \\
X_{it} &= \rho X_{i,t-1} + \tau_{it} + \epsilon_{it}
\end{align*}
\]

The estimation findings following this system of equations are presented in Table 2. As shown in the results, there is evidence of bias in the estimators in the model estimated by pooled OLS, RE, and FE due to the inability of the models to correct for the existence of the endogeneity problem. In contrast, the estimators of GMM-Diff and GMM-Sys present substantially satisfactory findings as projected by the true value.
Table 2

Summary of the estimator performance under the correlation between the regressor and unobserved heterogeneity cases

<table>
<thead>
<tr>
<th>True value: $\beta = 0.7789$</th>
<th>Parameter</th>
<th>Mean</th>
<th>SD</th>
<th>Minimum</th>
<th>Maximum</th>
<th>RMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pooled OLS</td>
<td>$\beta$</td>
<td>0.3669</td>
<td>0.07073</td>
<td>0.3599</td>
<td>0.3738</td>
<td>0.1697</td>
</tr>
<tr>
<td>RE</td>
<td>$\beta$</td>
<td>0.3670</td>
<td>0.1358</td>
<td>0.3337</td>
<td>0.3803</td>
<td>0.1697</td>
</tr>
<tr>
<td>FE</td>
<td>$\beta$</td>
<td>0.3670</td>
<td>0.1341</td>
<td>0.3339</td>
<td>0.3802</td>
<td>0.1697</td>
</tr>
<tr>
<td>GMM-Diff</td>
<td>$\beta$</td>
<td>0.5465</td>
<td>1.5486</td>
<td>0.3947</td>
<td>0.6983</td>
<td>0.0501</td>
</tr>
<tr>
<td>GMM-Sys</td>
<td>$\beta$</td>
<td>0.4275</td>
<td>0.4039</td>
<td>0.3879</td>
<td>0.4671</td>
<td>0.1235</td>
</tr>
</tbody>
</table>

Note: SD, standard deviation; min, minimum; max, maximum; RMSE, root mean square error.
Source: Author’s estimations (2022).

Temporal persistence of the explained variable

Under this scenario, we present a specific case to highlight the importance of including a dynamic term in the model when the dependent variable is greatly persistent. However, several empirical studies on the spatial concepts carry out model estimation in their static form, where ($\alpha = 0$), resulting in an inadequate specification of the empirical model, as represented below:

$$Y_{it} = \alpha Y_{it-1} + \beta X_{it} + \eta_i + \mu_{it}$$

$$X_{it} = \rho X_{it-1} + \epsilon_{it}$$

Table 3

Summary of the estimator performance under temporal persistence of the explained variable case

<table>
<thead>
<tr>
<th>True values: $\alpha = -6.3092$; $\beta = 0.7789$</th>
<th>Parameter</th>
<th>Mean</th>
<th>SD</th>
<th>Minimum</th>
<th>Maximum</th>
<th>RMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stat-Pooled OLS</td>
<td>$\beta$</td>
<td>0.3669</td>
<td>0.1345</td>
<td>0.3537</td>
<td>0.3801</td>
<td>0.1697</td>
</tr>
<tr>
<td>Dyn-Pooled OLS</td>
<td>$\alpha$</td>
<td>–6.8919</td>
<td>13.1402</td>
<td>–8.1836</td>
<td>–5.6003</td>
<td>0.3395</td>
</tr>
<tr>
<td>Dyn-FE</td>
<td>$\beta$</td>
<td>0.8225</td>
<td>0.8704</td>
<td>0.7392</td>
<td>0.9103</td>
<td>0.0019</td>
</tr>
<tr>
<td>Dyn-GMM-Diff</td>
<td>$\alpha$</td>
<td>–3.7740</td>
<td>71.0432</td>
<td>–10.7361</td>
<td>3.1882</td>
<td>6.4272</td>
</tr>
<tr>
<td>Dyn-GMM-Sys</td>
<td>$\beta$</td>
<td>0.6168</td>
<td>4.7372</td>
<td>0.1526</td>
<td>1.0811</td>
<td>0.0263</td>
</tr>
</tbody>
</table>

Note: Dyn, dynamic; Stat, static; SD, standard deviation; min, minimum; max, maximum; RMSE, root mean square error.
Source: Author’s estimations (2022).

The results presented in Table 3 show the temporal persistence estimation when the dependent variable’s dynamic term ($Y_{it-1}$), as part of the independent variables...
following Model (23). persists. As shown in Table 3, GMM-Diff tends to correctly estimate the coefficient estimate both as a static and dynamic variable, as the estimated parameters are nearly equal to the true value of the estimate, that is $\beta = 0.7779 \approx 0.7163 \approx 0.7789$. We do not include the estimates of RE and GMM-Sys, as they provide similar results to FE and GMM-Diff, respectively.

**Feedback effects case**

In this case, we illustrate how feedback effects that flow from Y to X, represented by the term $(\theta \mu_{it-1})$, might impact the estimates. The findings of the estimation are presented in Table 4. The estimations follow the model stated as follows:

$$
Y_{it} = \alpha Y_{it-1} + \beta X_{it} + \mu_{it} \\
X_{it} = \rho X_{it-1} + \theta \mu_{it-1} + \epsilon_{it}
$$

(24)

As anticipated, the pooled OLS estimators closely match the estimators' true values since, as explained above, they do not rely on the strict exogeneity conditional assumption. Thus, this assumption is not violated by the dynamic endogeneity phenomenon. The same findings are reflected by the RE method. However, the FE estimator, which relies on the strict exogeneity conditional assumption, greatly underestimates the coefficients. As captured by Model (24), since this conditional assumption is directly violated, the coefficient estimates are inconsistently determined, and the most affected estimate is ($\alpha$). In contrast, the GMM-Diff and GMM-Sys models produce the most satisfactory results for both ($\alpha$) and ($\beta$). The results show that ($\alpha = -3.2045$; $\beta = 0.6451$) and ($\alpha = -6.6487$; $\beta = 0.8085$) for GMM-Diff and GMM-Sys, respectively. This satisfactory performance can be attributed to the fact that the models adopt the conditional assumption that X is a predetermined variable. We omit the results of the RE method, as it exhibits similar findings to those of the pooled OLS method. The results are presented in Table 4.

**Summary of the estimator under the feedback effects case**

<table>
<thead>
<tr>
<th>Model</th>
<th>Parameter</th>
<th>Mean</th>
<th>SD</th>
<th>Minimum</th>
<th>Maximum</th>
<th>RMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pooled OLS</td>
<td>$\alpha$</td>
<td>-6.8180</td>
<td>13.2638</td>
<td>-8.1178</td>
<td>-5.5181</td>
<td>0.2589</td>
</tr>
<tr>
<td></td>
<td>$\beta$</td>
<td>0.8198</td>
<td>0.8703</td>
<td>0.7346</td>
<td>0.9051</td>
<td>0.0017</td>
</tr>
<tr>
<td>FE</td>
<td>$\alpha$</td>
<td>-3.3259</td>
<td>80.3259</td>
<td>-11.1977</td>
<td>4.5459</td>
<td>8.9001</td>
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<tr>
<td></td>
<td>$\beta$</td>
<td>0.5870</td>
<td>5.3300</td>
<td>0.0646</td>
<td>1.1092</td>
<td>0.0368</td>
</tr>
<tr>
<td>GMM-Diff</td>
<td>$\alpha$</td>
<td>-3.2045</td>
<td>70.2976</td>
<td>-10.0936</td>
<td>3.6845</td>
<td>9.6392</td>
</tr>
<tr>
<td></td>
<td>$\beta$</td>
<td>0.6451</td>
<td>68.4062</td>
<td>-6.0586</td>
<td>7.3488</td>
<td>0.0179</td>
</tr>
<tr>
<td>GMM-Sys</td>
<td>$\alpha$</td>
<td>-6.6487</td>
<td>16.5075</td>
<td>-8.2663</td>
<td>-5.0310</td>
<td>0.1153</td>
</tr>
<tr>
<td></td>
<td>$\beta$</td>
<td>0.8085</td>
<td>1.0987</td>
<td>0.7009</td>
<td>0.9162</td>
<td>0.0009</td>
</tr>
</tbody>
</table>

*Note: SD, standard deviation; min, minimum; max, maximum; RMSE, root mean square error. Source: Author’s estimations (2022).*
Conclusion

Most econometric studies in the spatial context rely on regions' observational datasets to evaluate the cause–effect link between variables using linear regression methods. However, in almost every study, researchers experience the challenge of identifying and correcting the regressors’ endogeneity problem, which, if overlooked, leads to inconsistent results. Thus, this study aimed to review various applicability and specification forms of the dynamic panel model. Additionally, the study focused on estimation procedures for estimating the dynamic panel model in the case of endogeneity, unobserved heterogeneity, and feedback effect problems. The study evaluated the efficiency of each estimation model using the actual and simulated sample panel data for the dynamics of urban agglomeration and regional economic performance in Sub-Saharan Africa. The results shed light on the probable bias of the estimated coefficient estimates when the problems associated with measurement errors, feedback effects, or dynamic endogeneity are not sufficiently addressed. The study considered an integrated approach of using pooled OLS, RE, FE, GMM-Diff, and GMM-Sys procedures in carrying out estimations. The results show that the conventional method (pooled OLS, RE, and FE) estimators are inconsistent when endogeneity problems plausibly exist. Satisfactorily, the estimation procedures based on GMM methods, which include the assumption of sequential exogeneity of the independent variables, offer options that are ideal for overcoming all the problems listed. Specifically, the results show that GMM-Sys estimators are coupled with a low bias and high efficiency and are hence ideal for modeling spatial concepts with time-space dependences. However, the study is limited in some ways. First, the study is limited to only regression estimation methods, and the predictive efficiency may not apply to other estimation techniques of dynamic panel models, such as the vector autoregressive (VAR) model. Finally, the study is entirely limited to panel data and thus may not convey similar findings to those of strict time-series or cross-sectional studies. Future studies can focus on the time-series dimension in establishing the predictive efficiency of the regression methods. Additionally, future studies can ascertain whether other methods of estimating dynamic panel models point in the same direction as regression methods.
Appendix

Figure A1

Selected Sub-Saharan African countries
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